Calculating the Speed of Light

Steven (Niq) Cunningham^{*}

Department of Physics, University of Nebraska Omaha, Omaha, NE 68128

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Here we investigate how we stumbled upon the speed of light and the experiments that followed. Through these three case studies we find how advancements in experimentation paved the way for the precise calculation we use today. Within this article we will set up two experiments to help us answer the question: Can we generate an accurate measurement of the speed of light using equipment found here at the University of Nebraska Omaha?

Keywords: Light, Speed, Experiment

I. INTRODUCTION

The speed of light is the speed at which light waves propagate through different materials. It is considered a fundamental constant of nature that sets a universal speed limit over all material particles. In physics we use this measurement to precisely calculate large distances between astronomical objects all over the universe.

One of the first people to attempt calculating its value was an astronomer by the name of Galileo Galilei. His experiment was very simple, he and his assistant would each stand atop a hill with a covered lantern. Galileo would remove the cover from his lantern, and once his assistant saw the light, he would do the same. He was attempting to measure the time difference from when he revealed his lantern to when he saw his assistant's. Unfortunately, the time interval was way too small to calculate with any equipment from that time period. Thus, the speed of light was deemed infinite. For years it remained this way, failed experiments left and right had no hope of uncovering a finite value. Then, amidst the mid 1670's, an unexpected set of data emerged.

[1] In 1676, a Danish astronomer by the name of Ole Roemer was the first to measure the speed of light. At the time of it's discovery, Roemer was studying the time intervals between the eclipse of Jupiter and its moon Io in order to gather more information about its orbital period. Through years of research he stumbled across something very peculiar: The time intervals between expected eclipses of Io were shorter when Earth was at its closest point to Jupiter and longer when it was at it's farthest. Roemer realized that as the Earth traveled farther away from Jupiter, it took light longer to traverse the distance, meaning the speed of light in fact had a finite value. "Roemer estimated that light required twenty-two minutes to cross the diameter of the Earth's orbit. The speed of light could then be found by dividing the diameter of the Earth's orbit by the time difference". He calculated the speed of light to be 131,000 miles/sec. Close but no cigar for our friend Roemer. The true value comes down to 186,000 miles/sec but for the first ever calculation it was an amazing estimate.

[2] A couple years down the road we had our next attempt by a man named Hippolyte Fizeau in 1849. In order to produce a more precise estimate than Roemer, Fizeau "used a beam of light reflected from a mirror 8 kilometers away. The beam passed through the gaps between teeth of a rapidly rotating wheel. The speed of the wheel was increased until the returning light passed through the next gap and could be seen". Fizeau's method determined the speed of light to be 194,700 miles/sec beating Roemer's estimate and landing within 5 percent of the actual value!

[3] Lastly, coming along in 1862 we have our final contender Leon Foucault. Foucault believed he could produce an even better estimate by modifying Fizeau's 1849 experiment by replacing the rotating wheel with a mirror. His experiment generated a value of 185168.615 miles/sec. Which is extremely close the value we use today. In this paper we will be exploring two, smaller scaled experiments I put together to in order to determine if its possible to generate accurate measurements of the speed of light using equipment found here at the University of Nebraska Omaha.

II. EXPERIMENT (1)

During WWII, Louis Essen bounced beams of light through a fixed microwave cavity with known wavelengths to measure the speed of light based off of the light wave's frequency. By doing so he was able to match the wavelength of light to the fixed wavelengths within the system. By comparing the phase shift between the two he calculated its velocity at about 186282.112 miles per second. [4] " Sir Charles Galton Darwin, while supporting the work, observed that Essen would get the correct result once he had perfected the technique. Moreover, W.W. Hansen at Stanford University had used a similar technique and obtained a measurement which was more consistent with the conventional (optical) wisdom. However, a combination of Essen's stubbornness, his iconoclasm and his belief in his own skill at measurement (and a little help with calculations from Alan Turing) inspired him to refine his apparatus and to repeat his measurement in 1950, establishing a result of $299,792.5 \pm 1 \text{ km/s}^{\circ}$.

^{*} niqcunningham@unomaha.edu

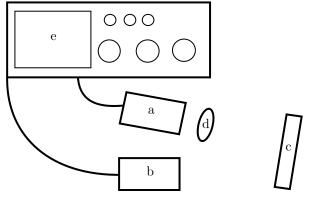


Fig 1: Top down view of the experimental setup. Laser (b) points towards mirror (c) which reflects back through the lens (d) into the photo detector (a). The waves of the laser as it leaves and returns are graphed seperately on the oscilloscope (e).

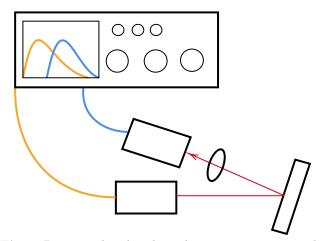


Fig 2: Diagram detailing how the system operates. The yellow wave represents the beam of light as it leaves the laser while the blue wave represents the light beam as it enters the photodetector.

A. Experimental Procedure

For this first measurement I recreated a simpler version of Essen's experiment using a pulsating laser, 50mm lens, mirror, photo-detector, and oscilloscope. Since light travels at such a fast speed, I needed to take measurements over a long distance and a nano-second scale in order to see a delay between the waves generated by the laser and photo-detector. So, I found a longest hallway I could on floor two of the Durham science center and got to work. I set up the laser, detector, and oscilloscope on top of a 3-tiered cart, then attached a small mirror with an adjustable bracket to a long metal pole 3 meters from the edge of the cart. Both the laser and detector were around 6.5cm back from the edge of the cart making the first round trip measurement 6.13m in total. The 50mm lens was placed a couple centimeters in front of the detector in order to re-focus the beam scattering as the distance of the mirror increased. As shown

in Fig 1, I connected the pulsating laser to channel 1 and the photo-detector to channel 2 of the oscilloscope and turned on the system. With the laser on, I adjusted the angle of the mirror to send the beam back towards the detector. From there I fine tuned the placement of the lens to condense the light to a nice point inside of the detector.

The oscilloscope measured the wavelength of the light beam as it left the laser and when it returned to the detector (as shown in Fig 2). It's scale was set to 100ns in order to clearly see the peaks of each wave. CH 1 displays a yellow wave for the beam as it leaves the laser. while CH 2 displays a blue wave for the beam as it returns the detector. For each measurement I recorded the difference in time between the peaks of these waves as the distance of the mirror increased. By doing so I was able to compare the phase shift between these two waves and use it to calculate the speed at which the beam was traveling. Starting at a distance of 3m from the cart, I let the oscilloscope gather information for around 10 seconds, paused the system, recorded the data, then increased the distance of the mirror. The cart and stands for the laser/detector were very sensitive to any small movements so instead of moving the cart, I moved the mirror back in increments of 2m all the way up to 17m (34.2m round trip from the laser to detector).

B. Measurements

As shown in the table below, I recorded the phase shift over eight different mirror positions. The data gathered shows a continual delay in readings from the photodetector as the distance of the mirror increased. Even though I recorded data at a 100ns scale, it was still a little difficult to find the exact peak of the detector so I had to take into account an error of around 4ns just to be safe.

| Speed of Light RAW DATA | | | |
|-------------------------|-----------|------------|-------|
| Beam Path | Error (m) | Phase | Error |
| Length (m) | | Shift (ns) | (ns) |
| 6.13 | 0.01 | 280 | 4 |
| 10.13 | 0.01 | 300 | 4 |
| 14.13 | 0.01 | 318 | 4 |
| 18.13 | 0.01 | 328 | 4 |
| 22.13 | 0.01 | 340 | 4 |
| 26.2 | 0.01 | 352 | 4 |
| 30.2 | 0.01 | 368 | 4 |
| 34.2 | 0.01 | 376 | 4 |

According to the table, as the mirror distance grows (AKA the length of the light beam), so does the time difference between the readings of the wave leaving the laser and returning to the photo-detector. As we should expect, this is because light has to travel farther to reach it's destination like we saw with Ole Roemer's experiment. A graphical representation of this data (Fig 3) shows a nice linear relationship between distance and time.

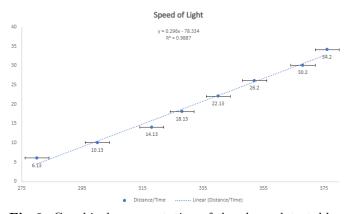


Fig 3: Graphical representation of the above data table with distance along the y-axis and the corresponding phase shift along the x-axis.

We know that c = distance (m) / time (ns) but in this case it won't be that easy. This equation only accounts for two data points, but we want to include every point we've gathered so far. To do this we need to find the best fit line over every point and use it's slope to calculate the speed of light. From the graph above we can clearly see that our best fit line has a slope of 0.296 with a coefficient of determination

$$R^2 = 0.989$$

Since we didn't account for nanoseconds within the graph itself, we'll have to apply it to the given slope to get our final answer.

$$0.296/10^9 = 2.96 * 10^8 m/s$$

Since our data points aren't exactly linear, we also have to note that our best fit line is going to have some uncertainty. Looking at the maximum and minimum slopes over the entire data set gives us an uncertainty value of 0.0129. With this taken into account we can confidently rewrite our answer as

$$(2.96\pm0.129) * 10^8 m/s$$

III. EXPERIMENT (2)

For my second experiment I calculated the speed of light through the use of frequency and wavelength.

$$c = vf$$

By interfering with the electromagnetic waves exchanged between the iOlab and it's dongle during normal periods of data acquisition, I was able to map out the shape of these waves. As shown in Fig 4, the iOlab emits electromagnetic waves in every direction while the dongle detects the superposition of the direct and reflected waves at its position. The waves emitted from the backside of the iOlab will reflect off of the aluminum foil surface back towards the dongle. As we move this surface farther away, we can map out a wave pattern within the RSSI signal strength graph. Note that the wave we are mapping out is directly correlated to the movement of the aluminum foil surface. Meaning that the only distance we need to worry about is the relative path length R:

$$R = L2 - L1 = 2D$$

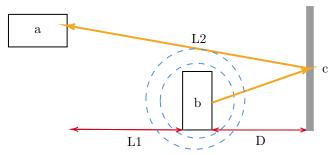


Fig 4: Diagram detailing the path of the electromagnetic waves exchanged between the dongle (a) and the iOlab body (b) as well as the waves intercepted by the aluminum foil (c). L1 is the distance between the dongle and iOlab. L2 Is the distance from the iOlab, to the foil, back to the dongle. D is the distance between the iOlab body and aluminum foil box.

A. Experimental Procedure

For this experiment I used an iOlab, aluminum foil, and the iOlab's measurement application (version 1.79.1602). After plugging the dongle into my laptop, I set it on top of two textbooks so the height of the dongle was greater than the height of the iOlab body (as shown in Fig.4). Next, I took the iOlab's storage box and wrapped it in aluminum foil, creating a reflective surface about 18cm tall and 17cm long that was placed at a starting distance of 10cm away from the main body. Within the application I used the RSSI signal strength graph to measure the interference pattern generated by the movement of the aluminum foil box as it intercepted the electromagnetic waves of the system. For each data point I moved the box back 5mm, recorded the signal strength for about 1 second and repeated this process 45 times in order to generate a good looking sin wave to work off of. Within the RSSI graph, we should expect to see adjacent minima when R has shifted by one wavelength. Since R = 2D, we will see adjacent minima when D has shifted by half of a wavelength.



Fig 5: RSSI signal strength graph representing all 45 data points. As you can see, the graphical representation of these points maps out a nice looking sin wave.

B. Measurements

Fig 5 shows the data recorded in a signal strength over time graph. After a total of 45 seconds we see that the data gathered has created a clear sin wave shape for the interference pattern. For this experiment I want my wavelength in terms of D. We know that the distance from one minima to another is about half of a wavelength, so in order to get a full wavelength I measured the distance over three total minima and found that time difference between the first and third minima was about 25 seconds

$$dt = 25s$$

Since the data was recorded in 1 second intervals, each being 5mm apart,

$$1s = 5mm$$

We can then multiply the difference in time (25 seconds) by 5mm to find the wavelength.

$$v = 25s * 5mm = 125mm = 0.125m$$

The iOlab states that it operates at 2.4Ghz giving us,

$$f = 2.4Ghz = 2.4 * 10^9 Hz$$

Now that we have values for the frequency of the system as well as wavelength, we can plug them back into our equation,

$$c = (2.4 * 10^9 Hz)(0.125m) = 3.00 * 10^8 m/s$$

Since this value was measured using the large dips of each minima and not each minima as a whole, it would be safe to assume an error value covering the entire length of each minima (roughly ± 1 second). Taking this error into account our new wavelength values become

$$v = 24s * 5mm = 120mm = 0.12m$$

 $v = 26s * 5mm = 130mm = 0.13m$

Thus giving us the following values for c,

$$c = (2.4 * 10^{9} Hz)(0.12m) = 2.88 * 10^{8} m/s$$
$$c = (2.4 * 10^{9} Hz)(0.13m) = 3.12 * 10^{8} m/s$$

With these possible errors in mind we can now rewrite our final answer as,

$$c = (3.00 \pm 0.12) * 10^8 m/s$$

IV. ANALYSIS

By graphically representing these values side by side with their errors (Fig 6), we can see a lot of overlap between the two points. Since I conducted two completely unrelated experiments to solve for the same value, we should expect the true value of the speed of light to fall somewhere in between this overlapping area. As of today the universal physical constant is equal to 299,792,458 m/s which falls right in between the results generated from both experiments.

$$(3.00\pm0.12) * 10^8 > 2.998 * 10^8 > (2.96\pm0.12) * 10^8$$

Thus proving that it is indeed possible to generate accurate measurements of the speed of light using equipment found here at the University of Nebraska Omaha.

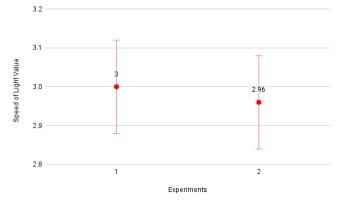


Fig 6: Graphical representation of both results detailing the overlap between these values.

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