The “complex form of calculus” was developed more than a century after Newton’s and Leibniz’s major contribution to real calculus. A central name is that of French mathematician Augustin Louis Cauchy. He decided that the complex plane was a bi—dimensional object and so, instead of integrating functions “along a line” (the real line of Calculus I), one can do it along curves in the plane, loops included. By using a definition of differentiability which is symbolically identical to that known from Calculus I, Cauchy noted that the properties of complex differentiable functions are very different in certain ways, from those of real ones. Most strikingly, by using an integral formula he discovered, Cauchy proved that a complex function differentiable once on an open set (i.e. a region of the plane containing none of its boundary points) is differentiable infinitely many times in that region. The situation in real calculus is totally different.

Indeed, the German mathematician Karl Weierstrass used the theory of infinite series of functions to construct a continuous nowhere differentiable function on the real line. Now, by the second form of the Fundamental Theorem of Calculus (as taught in Calculus I), the indefinite integral of Weierstrass’s function is a function differentiable once on the real line, but never twice.

Bernhard Riemann’s rigorous construction of the integral of classical calculus gave rigor immediately to the complex contour integral used by Cauchy. Furthermore, these two mathematicians tied real and complex differentiation showing that complex differentiation can be understood via a pair of real, partial differential equations, called nowadays “the Cauchy-Riemann” equations.

Pre-requisites: MATH 3230/MATH 8235 or equivalent.


Software: Any CAS installed in room 104 is accepted for computer projects (e.g. MAPLE, MATHEMATICA, MATLAB, and so on). No student purchase required.

Teaching presentation: Lecture

Grading: 20% homework and projects, 40% 3 in-class tests, 40% comprehensive final

A Weierstrass function done with a CAS by the UNO analysis students in 2018, as part of their course work.