

# Versorial Validation



A **quaternion** is a pretend-sum  $a + \mathbf{u}$  of a real number  $a$  and a 3D vector  $\mathbf{u}$  (called the scalar part and vector part respectively) which does not simplify.

The sum of quaternions is  $(a + \mathbf{u}) + (b + \mathbf{v}) = (a + b) + (\mathbf{u} + \mathbf{v})$ . The product of two vectors has scalar and vector parts given by dot product and cross product:  $\mathbf{u}\mathbf{v} = (-\mathbf{u} \cdot \mathbf{v}) + (\mathbf{u} \times \mathbf{v})$ . To multiply two generic quaternions, i.e.  $(a + \mathbf{u})(b + \mathbf{v})$ , we would use the distributive property (or “FOIL”).

The squared norm is  $|a + \mathbf{u}|^2 = |a|^2 + \|\mathbf{u}\|^2$  where  $\|\mathbf{u}\|$  is the vector norm, so

$$|w + x\mathbf{i} + y\mathbf{j} + z\mathbf{k}| = \sqrt{w^2 + x^2 + y^2 + z^2}.$$

**Problem.** Show  $|(a + \mathbf{u})(b + \mathbf{v})| = |a + \mathbf{u}||b + \mathbf{v}|$  (i.e. multiplicativity) using identities for dot products, cross products and vector norms.



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A photo of handwritten work is fine. You can also turn in solutions physically at the UNO math department’s mail room (located on the second floor of the Durham Science Center).