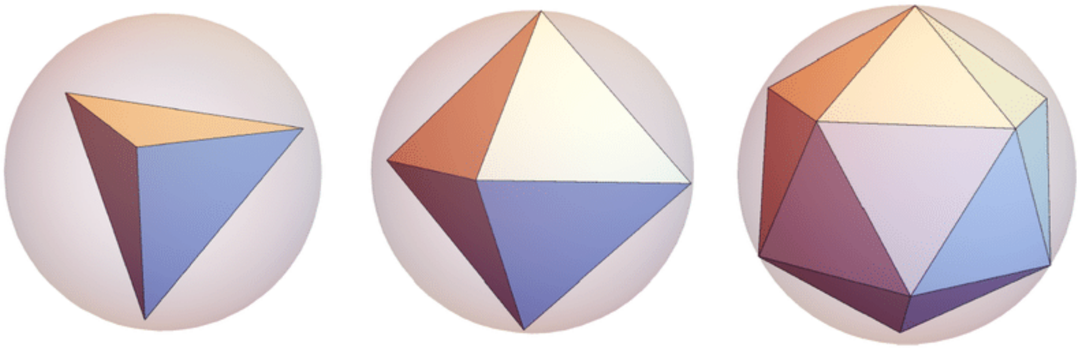
Striking Gold: Solution



One of the simplest triangulations involves inflating a tetrahedron until its edges become arcs on the sphere. According to MathWorld, its chromatic polynomial is $x(x-1)(x-2)(x-3)$, whose roots are not close enough.

Again by MathWorld, the octahedral graph has chromatic polynomial

$$x(x-1)(x-2)(x^3-9x^2+29x-32).$$

Plugging the last factor into Wolfram|Alpha, we find a root $x \approx 2.5466$.

The icosahedral graph has chromatic polynomial $x(x-1)(x-2)(x-3)$ times

$$x^8-24x^7+260x^6-1670x^5+6999x^4-19698x^3+3640x^2-40240x+20170.$$

Plugging the last factor into W|A again, we find a root $x \approx 2.6182$.

A theorem due to W. Tutte says spherical triangulations' chromatic polynomials tend to have a real root near $\varphi+1$, where $\varphi = \frac{1+\sqrt{5}}{2} \approx 1.618$ is the golden ratio. More precisely, if G is a planar graph with v vertices then

$$|P_G(\varphi+1)| \leq \varphi^{5-v}.$$