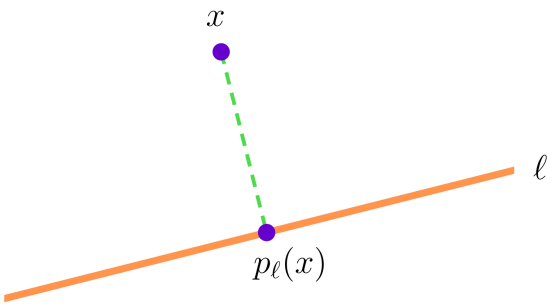
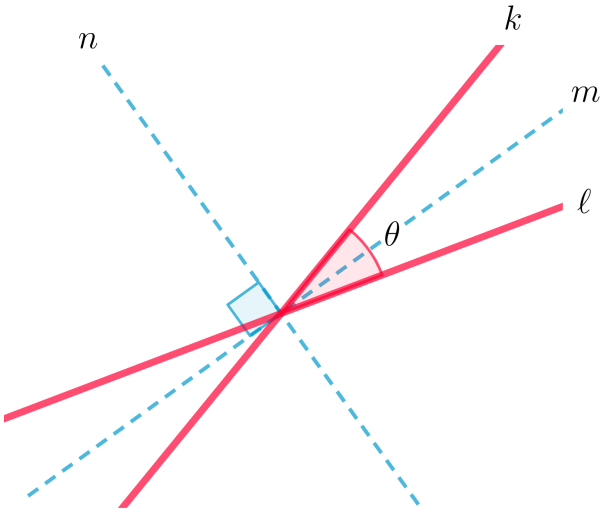
Projector Junction

For a line ℓ, the projector p_ℓ sends any point x to the nearest one $p_\ell(x)$ on ℓ:



The composition $p_k p_\ell$ of two projectors (as functions) is generally not a projector. However, the **symmetrization** $\frac{1}{2}(p_k p_\ell + p_\ell p_k)$ is a sum of orthogonal projectors, corresponding to the two bisectors m and n of k and ℓ:



Problem. Express $a(\theta)$ and $b(\theta)$ in terms of trigonometric functions of θ:

$$\tfrac{1}{2}(p_k p_\ell + p_\ell p_k) = a(\theta) p_m + b(\theta) p_n$$

Hint. For lines through an origin, projectors are linear operators.