Polarization

Suppose $\phi(\mathbf{a}, \mathbf{b}, \mathbf{c}, \mathbf{d})$ is a real-valued function of four 3D vectors satisfying:

- *Multilinearity*. If any three of the arguments are held fixed then ϕ is a linear function of the fourth argument, for example

$$\phi(\mathbf{a}_1 + \mathbf{a}_2, \mathbf{b}, \mathbf{c}, \mathbf{d}) = \phi(\mathbf{a}_1, \mathbf{b}, \mathbf{c}, \mathbf{d}) + \phi(\mathbf{a}_2, \mathbf{b}, \mathbf{c}, \mathbf{d})$$

- *Antisymmetry*. Swapping the first or second pair changes the sign:

$$\phi(\mathbf{b}, \mathbf{a}, \mathbf{c}, \mathbf{d}) = -\phi(\mathbf{a}, \mathbf{b}, \mathbf{c}, \mathbf{d}) = \phi(\mathbf{a}, \mathbf{b}, \mathbf{d}, \mathbf{c})$$

- *Symmetry*. Swapping the first pair with the second doesn't change it:

$$\phi(\mathbf{c}, \mathbf{d}, \mathbf{a}, \mathbf{b}) = \phi(\mathbf{a}, \mathbf{b}, \mathbf{c}, \mathbf{d})$$

- *Vanishing*. If the first pair equals the second pair, the result is zero:

$$\phi(\mathbf{a}, \mathbf{b}, \mathbf{a}, \mathbf{b}) = 0$$

Problem. Show $\phi(\mathbf{a}, \mathbf{b}, \mathbf{c}, \mathbf{d}) = 0$ for all $\mathbf{a}$, $\mathbf{b}$, $\mathbf{c}$, $\mathbf{d}$.

Submit your solution online by scanning QR code and filling out the form, or submit at

`sites.google.com/unomaha.edu/unopow`

A photo of handwritten work is fine. You can also turn in solutions physically at the UNO math department's mail room (located on the second floor of the Durham Science Center).