

Odd One Out: Solution

Suppose for the sake of contradiction a prime power p^v evenly divided only one of the denominators. It doesn't matter which one, so just say it was x .

This means p can only be a factor at most $v - 1$ times in the other denominators, so if we multiply the equation by p^{v-1} and move the other two fractions to the other side we get a new equation like the following:

$$\frac{A}{X} = \frac{B}{Y} + \frac{C}{Z}$$

We only multiplied by p^{v-1} , so the denominator X is still divisible by p . On the other hand, the new denominators Y and Z no longer have p as a factor (if they ever did to begin with...), which means when we add the fractions on the right, the new denominator (after simplifying) will be a factor of YZ , and so the denominator on the right will not be divisible by p :

$$\frac{A}{X} = \frac{D}{W}$$

In summary, when the fractions on both sides are in lowest terms, the left denominator is divisible by p but the right one isn't. This is a contradiction!