Joker's Wild: Solution

Suppose a player's current score is X. Let $c_1, \cdots, c_{k-1}, J$ be the cards they haven't drawn yet. The expected value of the next hand is then

$$E = \frac{c_1 + \cdots + c_{k-1} - X}{k}$$

If the Joker is drawn next then the value of the next hand is $-X$, since the score drops to 0. Note that X is the sum of all the cards that have been drawn so far, so $(c_1 + \cdots + c_{k-1}) + X = 1 + 2 + \cdots + 10 = 55$. Therefore,

$$E = \frac{(55 - X) - X}{k}$$

The expected value of the next hand is positive precisely when $55 - 2X > 0$, or $X < 27.5$. Thus we ought to set $S = 28$.