

Interesting Asymptotic



The constant e is often defined by the *interest* formula $e := \lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^n$.

Problem. What values of a and b make $1 + \frac{a}{n} + \frac{b}{n^2}$ the best possible approximation to $\frac{1}{e}\left(1 + \frac{1}{n}\right)^n$ as $n \rightarrow \infty$? We may define a for instance by

$$a = \lim_{n \rightarrow \infty} n \left[\frac{1}{e} \left(1 + \frac{1}{n}\right)^n - 1 \right].$$

Hint. Consider the Newton-Mercator series for $\ln\left(1 + \frac{1}{n}\right)$. (Look it up!)



Submit your solution online by scanning QR code and filling out the form, or submit at

sites.google.com/unomaha.edu/unopow

A photo of handwritten work is fine. You can also turn in solutions physically at the UNO math department's mail room (located on the second floor of the Durham Science Center).