Heat of Battle: Solution

The idea is to overlay all ship configurations together to get a **heat map** which for each cell counts how many configurations have a ship occupying it.

Wherever a 3-tile ship is, we can rotate or flip the board so that it is either vertical in the top left corner, or just to the right of that. In both cases, determine how many 2-tile ship placements cover the other cells (the yellow values $1, 2, 3, 4$ below). Each configuration, the 2-tile ship contributes $+2$ to the yellow total, so the number of configurations with the 3-tile ships is the yellow totals divided by 2 (the yellow-orange values $18, 15$).

18	2	3	2
18	3	4	3
18	3	4	3
1	3	3	2

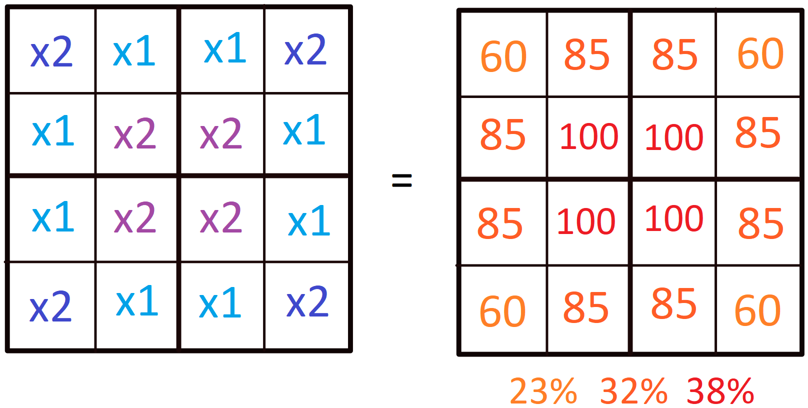
&

1	15	2	2
2	15	3	3
2	15	3	3
2	2	3	2

For both arrays above, there are a total of four rotated versions of the array, and then four more arrays that can be obtained by flipping those rotated versions. Instead of writing down sixteen arrays and then adding sixteen numbers for each of the sixteen cells, we can see what rotating and flipping does to individual cells. For example, any one corner cell when rotated and flipped lands at each of the four corner cells twice.

Considering this for all cells (or even just for three appropriate cells in a corner 2×2 block), we fill in the grid with sky, blue, and purple multipliers shown in the next figure. After adding the yellow and yellow-orange grids above together, we must apply the multipliers and then sum all the values in cells of a given multiplier color to get our final heatmap.

For example, after adding the grids, the corners clockwise from the top left are $18 + 1 = 19$, $2 + 2 = 4$, $2 + 2 = 4$, and $1 + 2 = 3$; doubling these values gives 38, 8, 8, and 6; adding these values gives $38 + 8 + 8 + 6 = 60$.



Each cell counts how many configurations have a ship occupying it, but to get probabilities we need to know how many configurations there are in total; each configuration contributes +5 tiles to the total sum of orange-red values above, so if we add all these orange-red values and divide by 5 we get a total of $4(60+2\cdot 85+100)/5 = 264$ configurations. Dividing the orange-red values by 264 gives the probabilities listed below the grid above.

Computing the heatmap for a larger battleship grid using more pieces of more shapes would require a computer.

The video game *The Legend of Zelda: The Wind Waker* includes an 8×8 battleship minigame, “Sploosh Kaboom,” which was once the bane of speedrunners because the game generates the ship configuration randomly. (Speedrunning is a competetive activity where very capable players beat video games, or certain levels in them, under certain conditions as fast as possible.)

A breakthrough occured after an app was created which not only uses a heatmap to show where the most probable hits are, but *updates* the heatmap after each hit or miss by throwing out all configurations inconsistent with the hit or miss. This is effectively an example of a **Bayesian search**.

Bayesian statistics has been applied successfully to real life searches where traditional searches failed (with cost-benefit predictions to boot): many lost ships and flights have been found from the ocean depths, including even a lost hydrogen bomb. The U.S. Coast Guard and U.S. Air Force adopted it for use in search and rescue operations after its success.