Arithmetic Jenga: Solution

Suppose n has the desired property. This means, in particular, for every prime factor p of n, the number $p-1$ must be noncomposite. This is true for $p=2$, but for $p>2$ the number $p-1$ is even, so unless $p-1=2$ that would mean $p-1$ is composite, a contradiction. Therefore, the only primes that may appear in n's prime factorization are 2 and 3.

Write $n=2^a 3^b$. Each of $2-1, 2^2-1, \cdots, 2^a-1$ must be noncomposite. We can check that $2-1, 2^2-1, 2^3-1$ are noncomposite but $2^4-1=3\cdot 5$ is composite, so $a \leq 3$. Similarly, $3-1$ is noncomposite but 3^2-1 is composite, so $b \leq 1$. The largest candidate is $n=2^3\cdot 3=24$. We can check:

d	24	12	8	6	4	3	2	1
$d-1$	23	11	7	5	3	2	1	0

By inspection we find that for every divisor d of 24 other than 1, the number $d-1$ is noncomposite, so $n=24$ has the desired property. Similarly, every other number of the form $n=2^a3^b$ with $a\leq 3$ and $b\leq 1$ has the property as well, and these are precisely the other divisors of 24 (listed above).