42: Solution

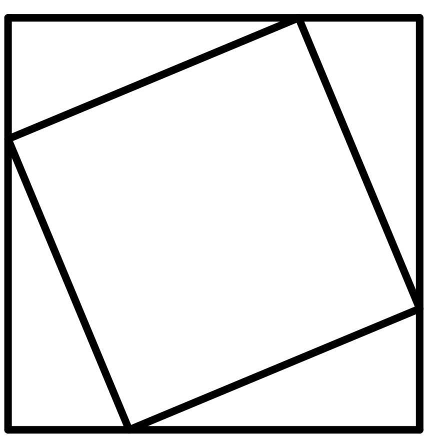
Each triangle has one 1, two 2s and three 3s. Thus, if all the numbers in a triangle are added together, the result is $1^2 + 2^2 + 3^2$. There are three triangles, so adding all the numbers up leaves us with $3(1^2 + 2^2 + 3^2)$.

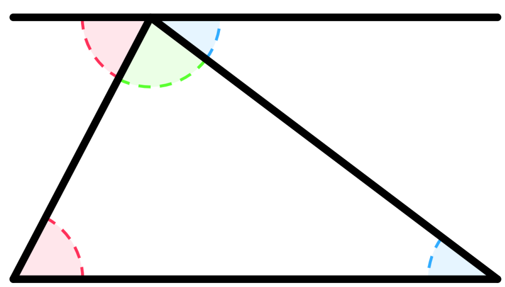
On the other hand, instead of summing the numbers within a triangle first, we could instead add them *across* triangles. In the top position, we get $1 + 2 \times 3$. Moving from one position to another either adds 1, does nothing, or subtracts 1 - thus, the sum of the three numbers in a given position is constant! The first row has 1 position, the second row 2 positions, and the third row 3 positions, so the sum of all the numbers is $1+2+3$ times $1+2\times 3$.

The general version of this has three triangles with n rows, yielding

$$3(1^2 + 2^2 + \cdots + n^2) = (1 + 2 + \cdots + n)(1 + 2n)$$

This is an example of a so-called **proof without words**: a picture which, if studied closely, can reveal a complete explanation of an interesting mathematical fact. Ideally, these proofs are supposed to be "self-evident" from the pictures, however realistically some thought, guidance, or mathematical background is still often necessary for real understanding.





Can you see $a^2 + b^2 = c^2$ or $A + B + C = 180°$ above? Deeper facts may also have proof without words, e.g. Dandelin spheres or Monge's Theorem.