Problem of the week #4

Due February 14th

The Fibonacci word sequence proceeds as follows:

$$\texttt{A, AB, ABA, ABAAB, ABAABABA, } \cdots$$

Each string is the concatenation of the previous two. Denote the nth word in the sequence by $S_n(\texttt{A}, \texttt{B})$, so for instance $S_3(\texttt{A}, \texttt{B}) =$`ABA`. Note the number of `A`'s in $S_n(\texttt{A}, \texttt{B})$ is F_{n-1}, and the number of `B`'s is F_{n-2}, where F_n denotes the nth Fibonacci number.

Problem. Suppose X and Y are noncommuting variables satisfying the relation $YX = qXY$, where q commutes with both X and Y.

The Fibonacci word $S_n(X, Y)$ will simplify to $q^{G(n)}X^{F_n}Y^{F_{n-1}}$ for some exponents $G(n)$ depending on n. Express $G(n) - G(n-1) - G(n-2)$ in terms of Fibonacci numbers, with explanation.

- Partial credit may be given for partial answers.
- Each POW will be due the following week at 1pm.
- Questions? Email: `bthorner@unomaha.edu`
- Submit solutions to (above email), DSC 210, or DSC 203.
- POWs, solutions, backgrounds, leaderboard available at

`https://www.unomaha.edu/college-of-arts-and-sciences/mathematics/student-opportunities/pow.php`