The Fibonacci word sequence proceeds as follows:

\[ A, AB, ABA, ABAAB, ABAABABA, \ldots \]

Each string is the concatenation of the previous two. Denote the \( n \)th word in the sequence by \( S_n(A, B) \), so for instance \( S_3(A, B) = ABA \). Note the number of \( A \)'s in \( S_n(A, B) \) is \( F_{n-1} \), and the number of \( B \)'s is \( F_{n-2} \), where \( F_n \) denotes the \( n \)th Fibonacci number.

**Problem.** Suppose \( X \) and \( Y \) are noncommuting variables satisfying the relation \( XY = qXY \), where \( q \) commutes with both \( X \) and \( Y \).

The Fibonacci word \( S_n(X, Y) \) will simplify to \( q^{G(n)} X^{F_n} Y^{F_{n-1}} \) for some exponents \( G(n) \) depending on \( n \). Express \( G(n) - G(n-1) - G(n-2) \) in terms of Fibonacci numbers, with explanation.

- Partial credit may be given for partial answers.
- Each POW will be due the following week at 1pm.
- Questions? Email: bthorner@unomaha.edu
- Submit solutions to (above email), DSC 210, or DSC 203.
- POWs, solutions, backgrounds, leaderboard available at