Solution to Problem ◇–9

Problem: *Let T be a triangle with sides 7, 8 and 9. (It is a solid triangle consisting of both interior and boundary points.)*

Find the perimeter of a planar figure formed by all points with distance at most 1 from the triangle T.

Solution. The figure described in the problem is the union of 7 pieces:
- the triangle T,
- three rectangles with sides 7×1, 8×1 and 9×1, adjacent to the sides of T of the length 7, 8, and 9, respectively,
- three circle sectors of radius 1 adjacent to each vertex. The sector adjacent to vertex with inside angle of measure α has angular measure of

$$2\pi - \left(\frac{\pi}{2} + \frac{\pi}{2} + \alpha\right).$$

  Consequently the three circle sectors add up to a complete circle of radius 1 (as the sum of the inside angles of a triangle is π).

Therefore, the perimeter of our figure is

$$7 + 8 + 9 + 2\pi = 24 + 2\pi.$$

$\square$