Solution to Problem $\diamondsuit$–7

Problem: *Find all functions* $f:\mathbb{R}\longrightarrow\mathbb{R}$ *satisfying*

$$f(x+y)=f(2x)+f(3y) \qquad \text{for all real } x,y.$$

Solution. Suppose that a function f satsifies the conditions stated in the problem.

Fix $t\in\mathbb{R}$ for a moment. Considering $x=y=\frac{1}{3}t$ we have

$$f(\frac{2}{3}t)=f(\frac{1}{3}t+\frac{1}{3}t)=f(\frac{2}{3}t)+f(t),$$

and hence $f(t)=0$.

Since t above was arbitrary, we conclude that any function satisfying the demands of the problem must have the constant value 0. Finaly we note that the function $f_0(x)=0$ indeed has the required property:

$$f_0(x+y)=0=0+0=f_0(2x)+f_0(3y).$$

□

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