Solution to Problem $\diamondsuit$–4

Problem: *2019 digits, none of them 0, are randomly (and independently) generated. Find the probability that their product is divisible by 10.*

Solution. The number obtained as a product of *digits* $n_1, n_2, \ldots, n_{2019}$ is divisible by 10 if and only if

(A) at least one of the n_i's is even, and
(B) at least one of the n_i's equals 5.

Since the successive numbers are picked indepedently (with equal probability),

- the probability of the event "there are no even digits in the constructed sequence" is $(5/9)^{2019}$,
- the probability of the event "there is no 5 in the constructed sequence" is $(8/9)^{2019}$,
- the probability of the event "no there are no even digits and no 5 in the constructed sequence" is $(4/9)^{2019}$.

For events A and B, denoting by A' the complement of A, we have

$$P(A\cap B) = 1-P\Big((A\cap B)'\Big) = 1-P\Big(A'\cup B'\Big) = 1-\Big[P(A')+P(B')-P(A'\cap B')\Big].$$

Therefore, the probability in question is

$$1-(8/9)^{2019}-(5/9)^{2019}+(4/9)^{2019}.$$

□

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