Solution to Problem ♢–3

Problem: *Find the maximum height above the x–axis of the cardioid $r = 1 + \cos(\theta)$.*

Solution. On the given cardioid,

$$x = (1 + \cos(\theta))\cos(\theta) \quad \text{and} \quad y = (1 + \cos(\theta))\sin(\theta).$$

The question is to find the maximum value $y_{\max}$ of y. Note that $y > 0$ is equivalent to $\sin(\theta) > 0$. From

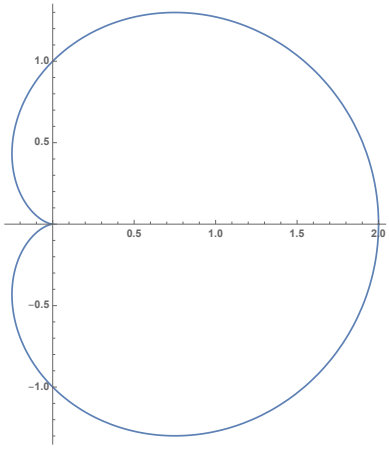
$$\frac{dy}{d\theta} = 2\cos^2(\theta) + \cos(\theta) - 1$$

we get that the critical numbers of the function $y = y(\theta)$ are the values of θ for which $\cos(\theta) = -1$ or $\cos(\theta) = \frac{1}{2}$. Since $y_{\max} > 0$ (and thus $\sin(\theta) > 0$) it follows that $\theta_{\max}$ giving the maximum value $y_{\max}$ of y satisfies

$$\sin(\theta_{\max}) = \sqrt{1 - \frac{1}{4}} = \frac{\sqrt{3}}{2}$$

and the maximum height equals

$$y_{\max} = (1 + \frac{1}{2})\frac{\sqrt{3}}{2} = \frac{3\sqrt{3}}{4}.$$



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