

Problem \diamond -8

Due in DSC 222 by 12 noon, **Friday, March 08, 2019**

Problem: For which values of a parameter a the equation

$$(a^2 + 1)(x^2 + y^2) - 2(a + 1)(x + y) + 2(1 + 2axy) = 0$$

has exactly one solution $(x, y) \in \mathbb{R}^2$?

RULES:

- The competition is open to all *undergraduate* UNO students and it is supervised by *Upper Curriculum Committee* of the Mathematics Department.
- Submit your solutions to Andrzej Rosłanowski in DSC 222 or to his mailbox.
- Every nontrivial step/claim in your solution must be justified. You may cite/quote a result from your textbook, past problems of the week and other widely available sources. In each case you have to give full reference.
- There are no partial credits, so rather err on the side of caution and provide more explanations than less. If you are not sure that your sources/references are appropriate, please include the complete relevant proofs from there.
- Your answers should be written clearly and legibly. We reserve the right to refuse grading your work if it is difficult to read it.
- The winners of Spring 2019 edition of POW will be determined at the end of the semester based on the number of correct solutions submitted.
- Problems will be posted by Friday 5pm and the solutions are due by the following Friday 12 noon.

PRIZES:

- Winners will receive books published by the American Mathematical Society. The titles actually awarded will be selected in cooperation with the awardees.
- Everybody scoring in the POW Competition qualifies for the grand finale:
 $\frac{\pi}{2}$ *Mathematical Competition.*