Solution to Problems ♠–9

Problem A: *Prove that for any positive integer n, there are infinitely many sequences of n consequtive composite positive integers.*

Answer: Let n be a positive integer.

The sequences $m, m+1, ..., m+n-1$, where $m = t(n+1)! + 2$, $t = 1, 2, \ldots$ are the sequences of n consequtive composite positive integers, because $i+2$ divides $m+i$ and $i+2 < m+i$, for $i = 0, 1, \ldots, n-1$, so $m+i$ are composite numbers.

CORRECT SOLUTIONS WERE RECEIVED FROM :

(1) BRAD TUTTLE

Problem B: *Find the sum:*

$$\ln\big(\tan 1^\circ\big) + \ln\big(\tan 2^\circ\big) + \ln\big(\tan 3^\circ\big) + ... + \ln\big(\tan 89^\circ\big).$$

Answer: We note that $\tan l^\circ \cdot \tan(90^\circ - l^\circ) = 1, \quad 1 \le l \le 89$. Thus, we have

$$2\sum_{l=1}^{89} \ln\big(\tan l^\circ\big) = \sum_{l=1}^{89} \Big(\ln\big(\tan l^\circ\big) + \ln\big(\tan(90^\circ - l^\circ)\big)\Big)$$

$$= \sum_{l=1}^{89} \ln\big(\tan l^\circ \cdot \tan(90^\circ - l^\circ)\big) = \sum_{l=1}^{89} \ln(1) = 0.$$

Hence

$$\ln\big(\tan 1^\circ\big) + \ln\big(\tan 2^\circ\big) + \ln\big(\tan 3^\circ\big) + ... + \ln\big(\tan 89^\circ\big) = 0.$$

CORRECT SOLUTIONS WERE RECEIVED FROM :

(1) ALI AL KADHIM — POW 9B: ♠
(2) CODY ANDERSON — POW 9B: ♠
(3) GAGE HOEFER — POW 9B: ♠
(4) BRAD TUTTLE — POW 9B: ♠