# Solution to Problems ♠–11

**Problem A:** *All the points of a circle are arbitrarily painted in two colors. Does there necessarily exist an isosceles triangle, whose vertices are all the same color, inscribed in the circle?*

**Answer:** Consider an inscribed regular pentagon. At least three of its vertices are on the same color because there are only two colors. Since any three vertices of a regular pentagon form an isosceles triangle, the positive answer to our question follows.

Correct solutions were received from :

(1) Brad Tuttle

**Problem B:** *A pentagon is such that each triangle formed by three adjacent vertices has area 1. Find its area.*

**Answer:** Let the pentagon be $\pentagon ABCDE$. Triangles $\triangle BCD$ and $\triangle ECD$ have the same area, so $B$ and $E$ are at the same perpendicular distance from the line extending $\overline{CD}$, so $\overline{BE}$ is parallel to $\overline{CD}$. The same applies to the other diagonals (each is parallel to the side with which it has no endpoints in common). Let $\overline{BD}$ and $\overline{CE}$ meet at X. Then $ABXE$ is a parallelogram, so Area$(\triangle BXE) =$ Area$(\triangle EAB) = 1$. Also

$$\text{Area}(\triangle CDX)+\text{Area}(\triangle EDX) = \text{Area}(\triangle CDX)+\text{Area}(\triangle BCX) = 1,$$

so Area$(\triangle EDX) =$ Area$(\triangle BCX)$. Put Area$(\triangle EDX) = x$. Then

$$\frac{|DX|}{|XB|} = \frac{\text{Area}(\triangle EDX)}{\text{Area}(\triangle BXE)} = \frac{x}{1},$$

and also

$$\frac{|DX|}{|XB|} = \frac{\text{Area}(\triangle CDX)}{\text{Area}(\triangle BCX)} = \frac{1-x}{x}.$$

Hence

$$x^2 + x - 1 = 0, \quad \text{and} \quad x = \frac{\sqrt{5}-1}{2}$$

(we know $x > 0$, so it cannot be the other root). Hence

Area$(\pentagon ABCDE) =$
Area$(\triangle EAB) +$ Area$(\triangle BXE) +$ Area$(\triangle BCD) +$ Area$(\triangle EDX) =$
$3 + x = \frac{\sqrt{5}+5}{2}$.

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(1) Brad Tuttle — POW 11B: ♠