Unequal Booty: Solution

[Four pirate version solution. Three pirates is guess-and-check.] Call the number of coins in the chest A, the number of coins the captain gets first B, the number of coins the second pirate gets first C, the number of coins the third gets first D, the number the swabbie gets first E, and the number of coins each pirate gets in the final handout F. Each division yields an equation:

$$\begin{aligned} A &= 1 + 3B \\ 2B &= 1 + 3C, \\ 2C &= 1 + 3D, \\ 2D &= 1 + 3E, \\ 2E &= 1 + 3F. \end{aligned}$$

A series of substitutions allows us to write

$$\begin{aligned} A &= 1 + 3B \\ &= 1 + \tfrac{3}{2}(1 + 3C) \\ &= 1 + \tfrac{3}{2}(1 + \tfrac{3}{2}(1 + 3D)) \\ &= 1 + \tfrac{3}{2}(1 + \tfrac{3}{2}(1 + \tfrac{3}{2}(1 + 3E))) \\ &= 1 + \tfrac{3}{2}(1 + \tfrac{3}{2}(1 + \tfrac{3}{2}(1 + \tfrac{3}{2}(3F)))). \end{aligned}$$

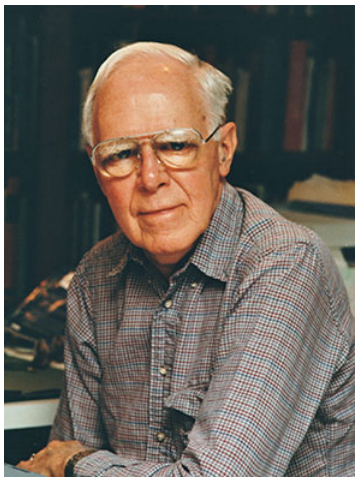
Multiplying by $2^4 = 16$ rids fractions, then find quotient/remainder of $\div 16$:

$$\begin{aligned} 16A &= 16 + 3(8 + 3(4 + 3(2 + 3(6F)))) \\ &= 16 + 24 + 46 + 54 + 243F \\ &= 130 + 243F \\ &= 16(8 + 15F) + (2 + 3F) \end{aligned}$$

Subtracting and factoring allows us to write

$$16(A - 8 - 15F) = 2 + 3F.$$

The larger F is, the larger the number of coins in the chest is. We seek the smallest value of F that makes $2 + 3F$ divisible by 16. Or, equivalently, the smallest multiple of 16 that is 2 greater than a multiple of 3. This last interpretation lends itself to a quick answer: 32 is the smallest such multiple of 16, yielding $F = 10$, and subsequently $E = 15$, $D = 23$, $C = 35$, $B = 53$ and the smallest number of coins in the chest is $A = 160$.



This is an adaptation of the *monkey and the coconuts* problem, the favorite problem of probably the most famous mathematical columnist of all, Martin Gardner, writer for the Mathematical Games column of the *Scientific American* magazine for a quarter century, and publisher of over a hundred books. Besides popularizing recreational mathematics, he was also an expert on Lewis Carroll, and founded the now Committee for Skeptical Inquiry (CSI) to combat pseudoscience.