Totally Tubular: Solution

Below are six inequivalent nonseparating loops in six different colors.



Slicing the double donut vertically between the left and right halves would separate it into two donuts; slicing it vertically the other way would yield the primary (blue, yellow, and red) colored loops; slicing it horizontally would yield the secondary (purple, green, and orange) colored loops.

Any loop on the surface can be tightened until it becomes a **geodesic** (a curve that minimizes surface-distance between any two sufficiently close points on it; in flat space, it is well-known the geodesics are just straight lines).



Maryam Mirzakhani won the Fields Medal in 2014 for her work on the dynamics and geometry of Riemann surfaces. In particular, she showed the number of geodesics of length $\leq L$ can be estimated similar to the prime numbers $\leq x$.

A corollary says nonseparating loops on a double torus outnumber separating ones six to one.

Photo from the August 2014 Seoul convention of the International Congress of Mathematicians (ICM), by Gert-Martin Greuel for the Mathematisches Forschungsinstitut Oberwolfach