Synchronicity: Solution

At 11:00pm the minute hand is $360°/12 = 30°$ clockwise from the hour hand, and at 11:15pm the minute hand is a little more than 90° clockwise from the hour hand, so our solution is somewhere between these two times.

Every minute that passes, the minute hand rotates $360°/60 = 6°$ forward, and the hour hand rotates $6°/12 = 0.5°$ forward. If m is the number of minutes after 11:00pm our solution, then $30 + 6m - 0.5m = 90$ is the angle between the hands, and solving yields $m = 60/5.5 \approx 11$, or 11:11pm.

A common superstition has it that 11:11 is connected to coincidence.

Mathematics is not without its share of numerology and mysticism. Take for instance this exchange (from *The Man Who Knew Infinity*, p312):

> Once, in the taxi from London, Hardy noticed its number, 1729. He must have thought about it a little because he entered the room where Ramanujan lay in bed and, with scarcely a hello, blurted out his disappointment with it. It was, he declared, "rather a dull number," adding that he hoped that wasn't a bad omen. "No, Hardy," said Ramanujan. "It is a very interesting number. It is the smallest number expressible as the sum of two cubes in two different ways."

Indeed, $1729 = 1^3 + 12^3 = 9^3 + 10^3$. There is a 1729-dimensional Fourier transform used in a so-called galactic algorithm for computing the product of two integers. It is the fastest-known algorithm to multiply numbers in the long run, but the "long run" means its optimal efficiency doesn't kick in until numbers start having around `2^1729^12` bits.

Nature's favorite number might just be 24. Because of its balance between high divisibility and smallness, it is also a favorite choice (along with its maximal divisor 12) of humans for systems that involve frequent division (such as timekeeping; there are 24 hours per day and 12 months per year). There are much deeper mathematical coincidences involving 24, though.

Some considerations show strings wiggling in 1D have ground state energy

$$\frac{1}{2}\big(1+2+3+4+\cdots\big) \;=\; -\frac{1}{24},$$

appropriately "regularized." In string theory, "strings" are actually 2D tubes, and them vibrating in $(24+2)$-dimensional spacetime leads to a ground state energy of $24(-\frac{1}{24}) = -1$, which turns out to explain why 26 is the only consistent number of dimensions for the theory.

The only time the sum of the first n squares is itself a square is

$$1^2+2^2+3^2+4^2+\cdots+23^2+24^2 \;=\; 70^2.$$

For this reason, the null vector $(0, 1, 2, \cdots, 24, 70)$ can be used to construct the Leech lattice Λ_{24}, a unique 24-dimensional crystal. The symmetry group of the string theory built on Λ is the Monster group M. The smallest nontrivial irreducible representation of M has dimension 196884. Seemingly completely unrelatedly, the first nontrivial coefficient of the Fourier expansion of the j-invariant is 196883. When McKay pointed out to Conway that

$$196884 = 196883 + 1,$$

Conway called it moonshine (i.e. "crazy"). This coincidence, and the area of math created in its wake to explain it, is known as Monstrous Moonshine.