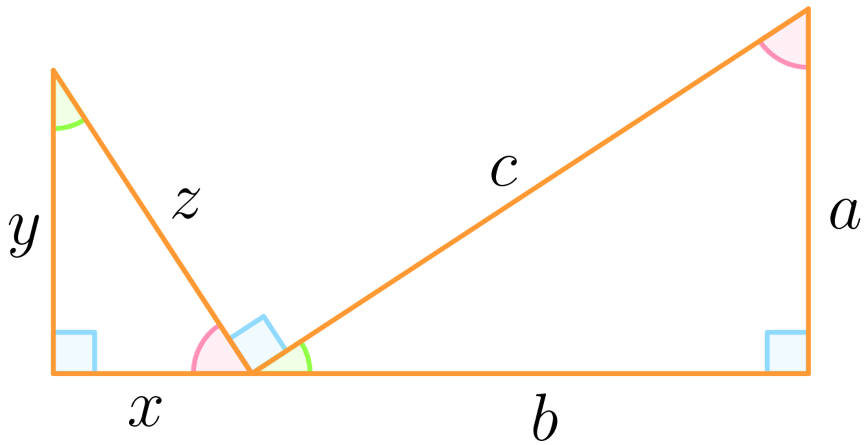
First Fold: Solution

Highlight and label the lower two triangles as follows:



The original top-left corner of the square is a right angle, which forces the two resulting triangles to be similar. The left side of the square is folded into $y + z = 1$ and the top side of the square becomes $c = 1$.

The Pythagorean theorem $x^2 + y^2 = z^2$ says $x^2 = z^2 - y^2 = (z - y)(z + y)$, which is just $z - y$. This establishes the linear system

$$\begin{cases} z + y = 1 \\ z - y = x^2 \end{cases} \implies \begin{cases} z = \frac{1}{2}(1 + x^2) \\ y = \frac{1}{2}(1 - x^2) \end{cases}$$

This follows from adding or subtracting, then halving, the equations.

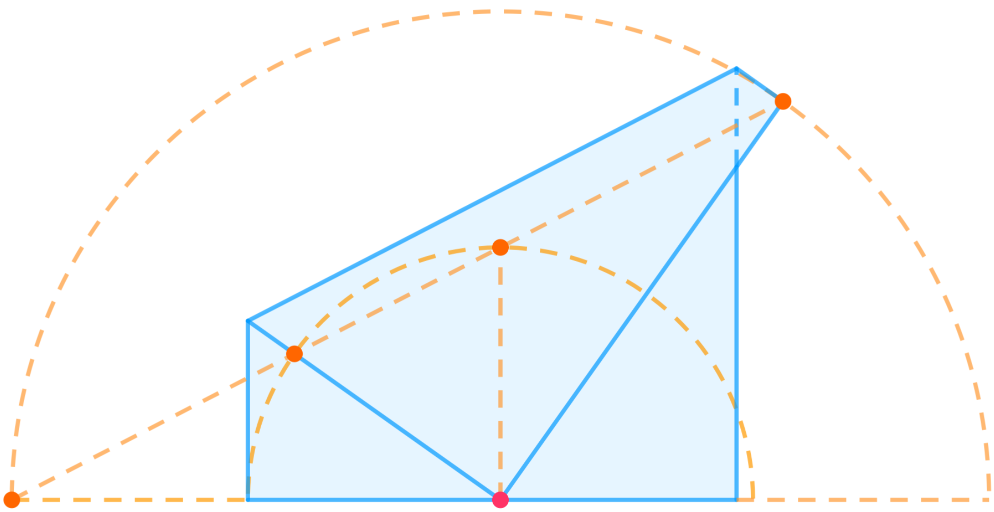
The similarity of the triangles means the proportions (ratios between corresponding sides) match. This is sometimes written $[a : b : c] = [x : y : z]$.

Given $c = 1$, equating ratios $[a : c] = [x : z]$ and $[b : c] = [y : z]$ yields

$$a = \frac{x}{z} = \frac{2x}{1 + x^2}, \qquad b = \frac{y}{z} = \frac{1 - x^2}{1 + x^2}.$$

In fact, these are the formulas for stereographic projection!

The picture of the fold with stereographic projection looks like:



This is called a **Haga fold** in mathematical paper folding.

By drawing lines to split the square into thirds, Haga folds can be used to solve the problems of doubling the cube and trisecting the angle, which are intractable with compass-and-straightedge constructions.