Like an Egyptian: Solution

Without loss of generality, $a < b < c$. Then $a \geq 2$ and $b \geq 3$. Furthermore,

$$\frac{1}{2} + \frac{1}{3} + \frac{1}{6} = 1,$$

which means for $c > 6$ we can maximize the sum using $a = 2$ and $b = 3$:

$$\frac{1}{a} + \frac{1}{b} + \frac{1}{c} \leq \frac{1}{2} + \frac{1}{3} + \frac{1}{7} = \frac{41}{42} < 1.$$

We maximize the above sum by maximizing the summands individually, which in turn we do by minimizing the denominators.

If $c = 6$, we know $(a, b) \neq (2, 3)$ which means we can similarly bound

$$\frac{1}{a} + \frac{1}{b} + \frac{1}{6} \leq \frac{1}{2} + \frac{1}{b} + \frac{1}{6} \leq \frac{1}{2} + \frac{1}{4} + \frac{1}{6} = \frac{11}{12} < \frac{41}{42}.$$

We can manually check all four possibilities for $\frac{1}{a} + \frac{1}{b} + \frac{1}{c}$ with $c < 6$:

$$\begin{aligned} \frac{1}{3} + \frac{1}{4} + \frac{1}{5} &= \frac{47}{60} < \frac{41}{42}, \\ \frac{1}{2} + \frac{1}{4} + \frac{1}{5} &= \frac{19}{20} < \frac{41}{42}, \\ \frac{1}{2} + \frac{1}{3} + \frac{1}{5} &= \frac{31}{30} > 1, \\ \frac{1}{2} + \frac{1}{3} + \frac{1}{4} &= \frac{13}{12} > 1. \end{aligned}$$

Thus we conclude $\frac{1}{2} + \frac{1}{3} + \frac{1}{7}$ is closest, being $\frac{1}{42}$ less than 1.