Child's Play: Solution

This is essentially a **tangram** - these are toy dissection puzzles, enjoyed by children, like jigsaw puzzles but with simple sold-colored polygon pieces and often with multiple different arrangements possible.

Indeed, this isn't just any tangram, it's the **Ostomachion** attributed to the mathematician Archimedes from Ancient Greece.

A natural question to ask is: *When is it possible to dissect one polygon and rearrange the pieces into another polygon?* Of course, the two polygons would need to have the same area. The Wallace–Bolyai–Gerwien theorem says this is not only necessary, it's actually sufficient too!

Hilbert posed the same question for 3D polyhedra. If two polyhedra can be dissected and rearranged into each other, the are called **scissors-congruent**.

Dehn answered the question by defining what we call the **Dehn invariant**, a kind of numerical signature a polyhedron has which does not change by dissection or rearrangement. In particular, the five Platonic solids have different Dehn invariants, so they are not scissors-congruent.

It turns out, two polyhedra are scissors-congruent if and only if they have both the same Dehn invariant *and* the same volume.