

## Problem of the week #5

Due October 4th

In four dimensions,  $\mathbb{R}^4$ , pick 24 points a distance of 1 from  $(0, 0, 0, 0)$ : the permutations of  $(\pm 1, 0, 0, 0)$  and  $\frac{1}{2}(\pm 1, \pm 1, \pm 1, \pm 1)$ . “Draw” an edge between any two of these points which are 1 unit apart, as per the distance formula (generalized from two and three dimensions).

**Problem.** For increasing levels of credit:

- (a) Give an example of a regular hexagon in this picture: list six of the vertices that are the correct distance of 1 and angle of  $60^\circ$  apart.
- (b) Find the number of these regular hexagons, with explanation.
- (c) Call a bundle any set of four of these regular hexagons which share no vertices or edges. Give an example of a bundle.

(*Bonus Credit.*) Find the number of bundles, with explanation.

*Hints.* Angles may be computed with the dot product just as they can in three dimensions. The most efficient calculations and readable explanations will use the phrase “by symmetry” multiple times! No symmetry argument will need justification, only correct usage.

- Partial credit may be given for partial answers.
- Each POW will be due the following week at 1pm.
- Questions? Email: [bthorner@unomaha.edu](mailto:bthorner@unomaha.edu)
- Submit solutions to (above email), DSC 210, or DSC 203.
- POWs, solutions, backgrounds, leaderboard available at

[https://www.unomaha.edu/college-of-arts-and-sciences/mathematics/student-opportunities/pow\\_solutions.php](https://www.unomaha.edu/college-of-arts-and-sciences/mathematics/student-opportunities/pow_solutions.php)