

## Problem of the week #11

Due November 15th

**Definition.** A whole number  $n$  is special if it is a factor of  $x^2 - 1$  for all integers  $x$  which do not share any factors with  $n$  besides  $\pm 1$ . For example, 2 is a factor of  $x^2 - 1$  for all odd numbers  $x$ , so 2 is special.

**Problem.** Find, with proof, all special numbers.

**Suggestion.** (i) Consider the smallest prime which is not a factor of  $n$ , and (ii) prove the lemma  $p_1 p_2 \cdots p_k > p_{k+1}^2$  (for sufficiently large  $k$ , where  $p_k$  is the  $k$ th prime) by induction using Bertrand's postulate.

(The postulate states there is always a prime between  $m$  and  $2m$ .)

- Partial credit may be given for partial answers.
- Each POW will be due the following week at 1pm.
- Questions? Email: [bthorner@unomaha.edu](mailto:bthorner@unomaha.edu)
- Submit solutions to (above email), DSC 210, or DSC 203.
- POWs, solutions, backgrounds, leaderboard available at

[https://www.unomaha.edu/college-of-arts-and-sciences/mathematics/student-opportunities/pow\\_solutions.php](https://www.unomaha.edu/college-of-arts-and-sciences/mathematics/student-opportunities/pow_solutions.php)