Solution to Problem ♣-4

Problem: Show that for every real number $a, b, c, d$ such that $c, d > 0$ we have

\[
\frac{(a + b)^2}{c + d} \leq \frac{a^2}{c} + \frac{b^2}{d}.
\]

Proof. Clearly $0 \leq (bc - ad)^2$, so we have

\[2abcd \leq b^2 c^2 + a^2 d^2.
\]

Consequently,

\[a^2 cd + b^2 cd + 2abcd \leq a^2 cd + b^2 cd + b^2 c^2 + a^2 d^2,
\]

and reordering both sides we obtain

\[(a + b)^2 cd \leq (a^2 d + b^2 c)(c + d).
\]

As $c, d > 0$ we immediately get

\[
\frac{(a + b)^2}{c + d} \leq \frac{a^2 d + b^2 c}{cd} = \frac{a^2}{c} + \frac{b^2}{d}.
\]

\[\square\]

There were six solutions submitted. Correct solutions were received from:

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