Solution to Problem ♣–4

Problem: *Show that for every real number a, b, c, d such that $c, d > 0$ we have*

$$\frac{(a+b)^2}{c+d} \leq \frac{a^2}{c} + \frac{b^2}{d}.$$

Proof. Clearly $0 \leq (bc - ad)^2$, so we have

$$2abcd \leq b^2c^2 + a^2d^2.$$

Consequently,

$$a^2cd + b^2cd + 2abcd \leq a^2cd + b^2cd + b^2c^2 + a^2d^2,$$

and reordering both sides we obtain

$$(a+b)^2cd \leq (a^2d + b^2c)(c+d).$$

As $c, d > 0$ we immediately get

$$\frac{(a+b)^2}{c+d} \leq \frac{a^2d + b^2c}{cd} = \frac{a^2}{c} + \frac{b^2}{d}.$$

□

There were six solutions submitted. Correct solutions were received from :

(1) David Cavanaugh — POW 4: ♣
(2) Jacob Cleveland — POW 4: ♣
(3) Ryan Fitzgibbons — POW 4: ♣
(4) Gage Hoefer — POW 4: ♣
(5) Brad Tuttle — POW 4: ♣
(6) Connor Wicks — POW 4: ♣