Solution to Problem ♣–2

Problem: *Let* $0 < p < 1$, $q = 1 - p$ *and* $x \geq 0$. *Show that*

$$pe^{xq} + qe^{-xp} \leq pe^{x^2q^2} + qe^{x^2p^2}$$

Proof. Let $0 < p < 1$, $q = 1 - p$ and $x \geq 0$. In Problem ♣–1 (which was due on 08/24/18) we showed that for every $t \in \mathbb{R}$ we have

$$e^t \leq e^{t^2} + t.$$

Therefore,

$$e^{xq} \leq e^{x^2q^2} + xq \quad \text{and} \quad e^{-xp} \leq e^{x^2p^2} - xp.$$

Consequently, as $0 < p, q$,

$$pe^{xq} \leq pe^{x^2q^2} + xpq \quad \text{and} \quad qe^{-xp} \leq qe^{x^2p^2} - xpq.$$

Adding these two inequalities side-by-side we get

$$pe^{xq} + qe^{-xp} \leq pe^{x^2q^2} + xpq + qe^{x^2p^2} - xpq = pe^{x^2q^2} + qe^{x^2p^2}.$$

□

There were 2 solutions submitted. Correct solution was received from :

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