Solution to Problem ♣–12

Problem: *Assume that Earth is a sphere with radius R. (In reality, R is about 6400 kilometers, but this has nothing to do with the solution of this problem. Express everything, including your answer, in terms of R.) A satellite has an elliptical orbit with the centre of Earth at one focus. The lowest point of the orbit is* $5R$ *above the surface of Earth, when the satellite is* directly above the North Pole. *The highest point of the orbit is* $11R$ *above the surface of Earth, when the satellite is* directly above the South Pole. *What is the height of the satellite above the surface of Earth, when the satellite is* directly above the equator *?*

Solution. Let the center of Earth (and a focus of the ellipse) be at the the point $(0, c)$, $c > 0$. Let the equation of the ellipse be

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1.$$

It is given that the vertices of the ellipse on the y–axis are

$$\big(0, (c + R) + 5R\big) \quad \text{and} \quad \big(0, (c - R) - 11R\big).$$

It follows that the length of the major axis on the y–axis is $2b = 6R + 12R = 18R$. Thus, $b = 9R$ and $c = b - 6R = 3R$. From $c^2 = b^2 - a^2$ we get that $a^2 = 72R^2$. Thus the equation of the ellipse is

$$\frac{x^2}{72R^2} + \frac{y^2}{81R^2} = 1.$$

The question is to evaluate the value of $|x|$ when $y = c = 3R$. From

$$\frac{x^2}{72R^2} + \frac{9R^2}{81R^2} = 1.$$

it follows that $|x| = 8R$. Consequently, the height of the satellite above the surface of Earth when it is directly above the equator is $7R$. □

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