# Solution to Problem ♣–11

**Problem:** *Show that for any odd positive integer we can always divide the set* $\{n, n+1, n+2, \ldots, n+32\}$ *into two parts, one with 14 numbers and one with 19, so that the numbers in each part can be arranged in a circle, with each number relatively prime to its two neighbours. [For example, for* $n = 1$*, arranging the numbers as* $1, 2, 3, \ldots, 14$ *and* $15, 16, 17, \ldots, 33$*, does not work, because 15 and 33 are not relatively prime.]*

*Solution.* We will give the decompositions considering 4 cases.

Case 1: $n \not\equiv 2 \mod 17$ and $n \not\equiv 0 \mod 13$.
Take $(n, n+1, \ldots, n+13)$ for the first circle. That certainly works since $n$ is not divisible by 13, since consecutive numbers are always relatively prime and any common divisor of $n$ and $n+13$ must also divide their difference 13. Then we take the second circle to be

$$\big(n+15, n+14, n+16, n+17, \ldots, n+32\big)$$

and note that any common factor of $n+14$ and $n+16$ must divide their difference 2, but $n$ is odd, so they are relatively prime. Similarly, any common factor of $n+15$ and $n+32$ must divide 17, but $n \not\equiv 2 \mod 17$, so they are relatively prime.

Case 2: $n \equiv 2 \mod 17$ and $n \not\equiv 0 \mod 13$.
Take $(n, n+1, \ldots, n+13)$ for the first circle and

$$\big(n+14, n+15, \ldots, n+29, n+30, n+32, n+31\big)$$

for the second circle. Any common factor of $n+30$ and $n+32$ must divide their difference 2, but $n$ is odd, so they are relatively prime. Similarly, any common factor of $n+14$ and $n+31$ must divide 17, but $n+14 \not\equiv 16 \mod 17$, so they are relatively prime.

Case 3: $n \not\equiv 16 \mod 17$ and $n \equiv 0 \mod 13$.
If $n$ is divisible by 13, then $n+19$ is not. We take

$$\big(n+19, n+20, \ldots, n+32\big)$$

for the first circle and $(n+1, n, n+2, n+3, \ldots, n+18)$ for the second circle.

Case 4: $n \equiv 16 \mod 17$ and $n \equiv 0 \mod 13$.
We take $\big(n+19, n+20, \ldots, n+32\big)$ for the first circle and

$$\big(n, n+1, \ldots, n+15, n+16, n+18, n+17\big)$$

for the second circle. □

Correct solution was received from :

(1) Zach Sabata — POW 11: ♣
(2) Brad Tuttle — POW 11: ♣