Solution to Problem ♣–10

Problem: *Find the domain and the absolute maximum value of the function*

$$f(x) = \sqrt{1 - \sqrt{2 - \sqrt{3 - x}}}$$

Solution. The domain D of the function f consists of all real numbers x for which

$$3 - x \geq 0, \text{ and } 2 - \sqrt{3 - x} \geq 0, \text{ and } 1 - \sqrt{2 - \sqrt{3 - x}} \geq 0.$$

The above inequalities are equivalent to

$$3 \geq x, \text{ and } 2 \geq \sqrt{3 - x}, \text{ and } 1 \geq \sqrt{2 - \sqrt{3 - x}},$$

respectively. Consequently,

$$\begin{aligned} D &= \left\{x \in \mathbb{R} : 3 \geq x, \text{ and } 4 \geq 3 - x, \text{ and } 1 \geq 2 - \sqrt{3 - x}\right\} \\ &= \left\{x \in \mathbb{R} : 3 \geq x, \text{ and } x \geq -1, \text{ and } 1 \leq \sqrt{3 - x}\right\} \\ &= \left\{x \in \mathbb{R} : 3 \geq x, \text{ and } x \geq -1, \text{ and } 1 \leq 3 - x\right\} \\ &= [-1, 2] \end{aligned}$$

The function f is continuous on $[-1, 2]$ and differentiable on $(-1, 2)$. Moreover, using the Chain Rule, we find that

$$f'(x) = \frac{1}{2\sqrt{1 - \sqrt{2 - \sqrt{3 - x}}}} \cdot \frac{-1}{2\sqrt{2 - \sqrt{3 - x}}} \cdot \frac{-1}{2\sqrt{3 - x}} \cdot (-1) < 0$$

for all $x \in (-1, 2)$. Therefore, f is strictly decreasing on its domain $[-1, 2]$ and its absolute maximum value is $f(-1) = 1$. □

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