

Problems ♣–14/15

Due in DSC 222 by 12 noon, **Friday, November 30, 2018**

Problem 14: Let $\theta > 0$. Show that

$$\frac{\sin(\theta)}{\theta} = \prod_{n=1}^{\infty} \cos\left(\frac{\theta}{2^n}\right).$$

Problem 15: Show that

$$\frac{2}{\pi} = \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{2+\sqrt{2}}}{2} \cdot \frac{\sqrt{2+\sqrt{2+\sqrt{2}}}}{2} \cdot \frac{\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2}}}}}{2} \cdots$$

RULES:

- The competition is open to all *undergraduate* UNO students and it is supervised by *Upper Curriculum Committee* of the Mathematics Department.
- Submit your solutions to Andrzej Rosłanowski in DSC 222 or to his mailbox.
- Every nontrivial step/claim in your solution must be justified. You may cite/quote a result from your textbook, past problems of the week and other widely available sources. In each case you have to give full reference.
- There are no partial credits, so rather err on the side of caution and provide more explanations than less. If you are not sure that your sources/references are appropriate, please include the complete relevant proofs from there.
- Your answers should be written clearly and legibly. We reserve the right to refuse grading your work if it is difficult to read it.
- The winners of Fall 2018 edition of POW will be determined at the end of the semester based on the number of correct solutions submitted.
- Problems will be posted by Friday 5pm and the solutions are due by the following Friday 12 noon.

PRIZES:

- Winners will receive books published by the American Mathematical Society. The titles actually awarded will be selected in cooperation with the awardees.
- Everybody scoring in the POW Competition qualifies for the grand finale: $\frac{\pi}{2}$ *Mathematical Competition*.