Problem 4-13

Due in DSC 222 by 12 noon, Friday, November 16, 2018

Problem: Show that for any positive integer n and $\theta > 0$ we have

$$\sin(\theta) = 2^n \cdot \sin\left(\frac{\theta}{2^n}\right) \cdot \cos\left(\frac{\theta}{2}\right) \cdot \cos\left(\frac{\theta}{4}\right) \cdot \cos\left(\frac{\theta}{8}\right) \cdot \ldots \cdot \cos\left(\frac{\theta}{2^n}\right).$$

Rules:

- The competition is open to all *undergraduate* UNO students and it is supervised by *Upper Curriculum Committee* of the Mathematics Department.
- Submit your solutions to Andrzej Rosłanowski in DSC 222 or to his mailbox.
- Every nontrivial step/claim in your solution must justified. You may cite/quote a result from your textbook, past problems of the week and other widely available sources. In each case you have to give full reference.
- There are no partial credits, so rather err on the side of caution and provide more explanations than less. If you are not sure that your sources/references are appropriate, please include the complete relevant proofs from there.
- Your answers should be written clearly and legibly. We reserve the right to refuse grading your work if it is difficult to read it.
- The winners of Fall 2018 edition of POW will be determined at the end of the semester based on the number of correct solutions submitted.
- Problems will be posted by Friday 5pm and the solutions are due by the following Friday 12 noon.

PRIZES:

- Winners will receive books published by the American Mathematical Society. The titles actually awarded will be selected in cooperation with the awardees.
- \bullet Every body scoring in the POW Competition qualifies for the grand finale:
 - $\frac{\pi}{2}$ Mathematical Competition.