

## Problem ♣-12

*Due in DSC 222 by 12 noon, Friday, November 09, 2018*

**Problem:** Assume that Earth is a sphere with radius  $R$ . (In reality,  $R$  is about 6400 kilometers, but this has nothing to do with the solution of this problem. Express everything, including your answer, in terms of  $R$ .) A satellite has an elliptical orbit with the centre of Earth at one focus. The lowest point of the orbit is  $5R$  above the surface of Earth, when the satellite is *directly above the North Pole*. The highest point of the orbit is  $11R$  above the surface of Earth, when the satellite is *directly above the South Pole*. What is the height of the satellite above the surface of Earth, when the satellite is *directly above the equator*?

### RULES:

- The competition is open to all *undergraduate* UNO students and it is supervised by *Upper Curriculum Committee* of the Mathematics Department.
- Submit your solutions to Andrzej Rosłanowski in DSC 222 or to his mailbox.
- Every nontrivial step/claim in your solution must be justified. You may cite/quote a result from your textbook, past problems of the week and other widely available sources. In each case you have to give full reference.
- There are no partial credits, so rather err on the side of caution and provide more explanations than less. If you are not sure that your sources/references are appropriate, please include the complete relevant proofs from there.
- Your answers should be written clearly and legibly. We reserve the right to refuse grading your work if it is difficult to read it.
- The winners of Fall 2018 edition of POW will be determined at the end of the semester based on the number of correct solutions submitted.
- Problems will be posted by Friday 5pm and the solutions are due by the following Friday 12 noon.

### PRIZES:

- Winners will receive books published by the American Mathematical Society. The titles actually awarded will be selected in cooperation with the awardees.
- Everybody scoring in the POW Competition qualifies for the grand finale:  
 $\frac{\pi}{2}$  *Mathematical Competition*.