

Problem ♡-2

Due in DSC 235 by 12 noon, Friday, September 22, 2017

Definition: A function $f : \mathbb{R} \rightarrow \mathbb{R}$ is *periodic* if there is a positive real number t such that

$$f(x) = f(x + t) \quad \text{for all } x \in \mathbb{R}.$$

Problem A: Suppose that a function $f : \mathbb{R} \rightarrow \mathbb{R}$ is nonconstant, periodic and has at least one continuity point. Prove that f has a smallest positive period, the so called *fundamental period*.

Problem B: Suppose that $f : \mathbb{R} \rightarrow \mathbb{R}$ is continuous and periodic with period $t > 0$. Prove that there is $x_0 \in \mathbb{R}$ such that

$$f\left(x_0 + \frac{t}{2}\right) = f(x_0).$$

RULES:

- The competition is open to all *undergraduate* UNO students.
- Please submit your solutions to Andrzej Roslanowski in DSC 235 or to his mailbox. (Needless to say, they should be written clearly and legibly.)
- The winners will be determined each semester based on the number of correct solutions submitted.
- Problems will be posted by Friday 5pm and the solutions are due by the following Friday 12 noon.

PRIZES:

- Winners will receive books published by the American Mathematical Society. The titles actually awarded will be selected in cooperation with the awardees.
- In Summer 2018, there is a research opportunity possibly that could lead to an Erdős Number (3 or possibly 2). Strong performance in POW is one of the crucial prerequisites.