# Problem ♡–2
*Due in DSC 235 by 12 noon, Friday, September 22, 2017*

**Definition:** A function $f : \mathbb{R} \longrightarrow \mathbb{R}$ is *periodic* if there is a positive real number $t$ such that
$$f(x) = f(x + t) \quad \text{for all } x \in \mathbb{R}.$$

**Problem A:** Suppose that a function $f : \mathbb{R} \longrightarrow \mathbb{R}$ is nonconstant, periodic and has at least one continuity point. Prove that $f$ has a smallest positive period, the so called *fundamental period*.

**Problem B:** Suppose that $f : \mathbb{R} \longrightarrow \mathbb{R}$ is continuous and periodic with period $t > 0$. Prove that there is $x_0 \in \mathbb{R}$ such that
$$f\left(x_0 + \frac{t}{2}\right) = f(x_0).$$