Problem ☀-9

Due in DSC 235 by 12 noon, Friday, November 10, 2017

**Problem A:** [From Brahmagupta (born c.598, died after 665)]
An old woman goes to market and a horse steps on her basket and crushes the eggs. The rider offers to pay for the damages and asks her how many eggs she had brought. She does not remember the exact number, but when she had taken them out two at a time, there was one egg left. The same happened when she picked them out three, four, five, and six at a time, but when she took them seven at a time they came out even. What is the smallest number of eggs she could have had?

**Problem B:** For a positive integer $m$ let $S(m)$ denote the sum of all digits of $m$ (in the decimal representation). Consider the sequence $(a_n)_{n=1}^{\infty}$ defined recursively as follows:

$$a_1 = 2017^{2017}, \quad a_{n+1} = S(a_n).$$

What is $a_{2017}$?

**Rules:**
- The competition is open to all undergraduate UNO students.
- Please submit your solutions to Andrzej Roslanowski in DSC 235 or to his mailbox. (Needless to say, they should be be written clearly and legibly.)
- The winners will be determined each semester based on the number of correct solutions submitted.
- Problems will be posted by Friday 5pm and the solutions are due by the following Friday 12 noon.

**Prizes:**
- Winners will receive books published by the American Mathematical Society. The titles actually awarded will be selected in cooperation with the awardees.
- In Summer 2018, there is a research opportunity possibly that could lead to an Erdős Number (3 or possibly 2). Strong performance in POW is one of the crucial prerequisites.