Problem –11
Due in DSC 235 by 12 noon, Friday, December 1, 2017

Problem A: Let \( (a_n)_{n=1}^\infty \) be a sequence of positive real numbers with all terms different from 1. Show that if \( \lim_{n \to \infty} a_n = 1 \), then
\[
\lim_{n \to \infty} \frac{\ln(a_n)}{a_n - 1} = 1.
\]

Problem B: Let \( (a_n)_{n=1}^\infty, (b_n)_{n=1}^\infty \) be sequences of positive real numbers such that
\[
\lim_{n \to \infty} a_n = a > 0 \quad \text{and} \quad \lim_{n \to \infty} b_n = b > 0.
\]
Suppose that \( p, q > 0 \) satisfy \( p + q = 1 \). Prove that
\[
\lim_{n \to \infty} (pa_n + qb_n)^n = a^p b^q
\]

Problem C: Find the limit of the sequence \( (a_n)_{n=1}^\infty \), where
\[
a_n = \left(1 + \frac{1}{n^2}\right) \cdot \left(1 + \frac{2}{n^2}\right) \cdot \ldots \cdot \left(1 + \frac{n}{n^2}\right), \quad \text{for } n = 1, 2, 3, \ldots
\]

Rules:

- The competition is open to all undergraduate UNO students.
- Please submit your solutions to Andrzej Roslanowski in DSC 235 or to his mailbox. (Needless to say, they should be written clearly and legibly.)
- The winners will be determined each semester based on the number of correct solutions submitted.
- Due to the Thanksgiving Day holidays, solutions to POW-11 are due on 12/01/17, two weeks after problems are posted. Happy Holidays!

Prizes:

- Winners will receive books published by the American Mathematical Society. The titles actually awarded will be selected in cooperation with the awardees.