Project Title:
The many ways to measure finite sets

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Description:
When dealing with various sets we may want to compare them and we may be interested in deciding which sets are larger than the other. The size (or "bigness") of a finite set can be measured/decided by counting how many elements it has. However, this way of deciding if a set is large assumes that all elements were created equal and it may be totally meaningless if the elements of our sets carry some structure. Therefore, in some situations, we may be interested in considering various *norms* on finite sets which measure how large various sets are, but which are very different from just counting the number of elements of a set. Let us give an example of such a norm.

Example [Bartoszyński–Shelah Norm]
Let I be a finite set (the intention is that it has a lot of elements) and let $[I]^2 \stackrel{\rm def}{=} \{(a,b) \in I \times I : a \neq b\}$. For a set $X \subseteq [I]^2$ we define $\|X\|^{\rm BS}$ as the smallest natural number k for which there are sets $A_1, B_1, A_2, B_2, \ldots, A_k, B_k$ such that

- *$A_i \cap B_i = \emptyset$ for $i = 1, \ldots, k$, and*
- *$X \subseteq \bigcup\limits_{i=1}^{k} (A_i \times B_i)$.*

The norms on finite sets have been of interest because thay may be used to construct various *infinite* objects. For example, the norm $\| \cdot \|^{\rm BS}$ defined above has been used to construct non–Hausdorff ultrafilters (see Bartoszyński and Shelah [1]). Another source of interest in norms on finite sets is the theory of forcing: every family of norms determines a forcing notion which can be used in independence proofs (see Rosłanowski and Shelah [2], [3]).

In this project we propose that the participating student engages in the following activities:

(A) Write a catalog of norms that have been already used in forcing constructions. This will require browsing through many research papers, but there will be no need to understand all of their content: the goal will be to find definitions of norms and possibly lists of their properties.

(B) Investigate the properties of the norms in the inventory and
  - write down the proofs of the known properties,

- investigate which norms can be characterized by their properties,
- search for new properties.

(C) Analyze known results in finite combinatorics, graph theory and likes to see if there are any new norms hidden behind them.

It is expected that the results of the work related to this project will include a survey of norms and possibly a research paper on them.

TIMETABLE:

January–March 2018	The student will read selected parts of research publications mentioned in this proposal.
April–May 2018:	The student will attempt to discover new norms on finite sets, and will investigate their properties.
May 2018:	Preparation of the final report and proposals for *NASA Nebraska Space Grant Fellowship* as well for *FUSE* grant.

REFERENCES

[1] Tomek Bartoszynski and Saharon Shelah. On the density of Hausdorff ultrafilters. In *Logic Colloquium 2004*, Lecture Notes in Logic, 29, pages 18–32. Association of Symbolic Logic, Chicago, 2008. Proceedings of the AMS. arxiv:math.LO/0311064.

[2] Andrzej Roslanowski and Saharon Shelah. Norms on possibilities I: forcing with trees and creatures. *Memoirs of the American Mathematical Society*, 141(671):xii + 167, 1999. arxiv:math.LO/9807172.

[3] Andrzej Roslanowski and Saharon Shelah. Sweet & Sour and other flavours of ccc forcing notions. *Archive for Mathematical Logic*, 43:583–663, 2004. arxiv:math.LO/9909115.