Title: Numerical methods for stochastic ordinary differential equations

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Student Name: This project is Available. If interested, contact me soon

Project Start Date: immediately **Project End Date:** 6/30/2017

Description: A stochastic differential equation (SDE) is a differential equation in which one or more of the terms is a stochastic process, thus resulting in a solution which is itself a stochastic process. SDE models play a prominent role in a range of application areas, including biology, physics, chemistry, epidemiology, mechanics, microelectronics, economics, and finance, etc., when uncertainties or random influences (called noises), are taken into account. Furthermore, SDE are used to model diverse phenomena such as fluctuating stock prices or physical system subject to thermal fluctuations. Typically, SDEs incorporate white noise which can be thought of as the derivative of Brownian motion (or the Wiener process); however, it should be mentioned that other types of random fluctuations are possible, such as jump processes.

In the deterministic setting, there exist many numerical methods to approximate the solution of a differential equation. However, in the stochastic setting, direct translations of these methods appear to be problematic. Numerical solution of SDEs is a young field relatively speaking. Almost all algorithms that are used for the solution of deterministic differential equations will work very poorly for SDEs, having very poor numerical convergence. Existing numerical methods for SDEs exhibit low orders of convergence. The discontinuous Galerkin (DG) finite element method is a promising avenue of research for developing higher order numerical methods, yet it is still in its infancy.

In this project, we propose to develop, implement, and analyze new high-order discontinuous Galerkin (DG) finite element method methods for solving SDEs arising in arising in various fields of science and engineering and present a rapid convergence for the solutions. The DG method have been successfully applied to many ordinary and partial deterministic differential equations arising from a wide range of applications. In this study, we propose to apply a stochastic analogue of the deterministic DG finite element method to SDEs. In particular, we will focus on application to SDEs arising in various applications, especially in fluid dynamics, biology, and finance. The goal is to construct efficient and stable numerical methods to solve nondeterministic SDEs. Full implementation details will be provided, including codes. Numerical experiments will be performed to show the convergence of the proposed schemes. If time allows, we will attempt to perform a theoretical error analysis of the method.

Prerequisites: Familiarity with differential equations, probability, and programming are required. You may use any programming language.

Tasks and Time Table:

By January 1st: Cover some existing numerical schemes that come from the Ito-Taylor expansion including their order of convergence. Implement some existing numerical methods and Finish the introduction/statement of problem.

By March 31st: Finish the motivation and formulation for the stochastic DG method.

By April 30th: Finish the implementation details section.

By June 30: Finish all codes to implement the proposed method. Perform the numerical experiments and write up the results. Finish writing the report (15-30 pages).