INTRODUCTION TO THE THEORY OF RECURSIVE FUNCTIONS

MATH/CSCI 4010/8016

Course Description:

This is a proof-oriented course presenting the foundations of Recursion Theory. We present the definition and properties of the class of primitive recursive functions, study the formal models of computation, and investigate partially computable functions, universal programs. We prove Rice's Theorem, the Recursion Theorem, develop the arithmetic hierarchy, demonstrate Post's theorem. Introduction to the formal theories of computability and complexity is also given.
3 credits

Prerequisites:

MATH 2230 or CSCI 3660 or instructor's permission

Overview of Content and Purpose of the Course:

Effectively calculable or computable functions have been studied by mathematicians since the early 1930s. They were systematically investigated by Kurt Godel, Alonzo Church, Alan Turing, Stephen Kleene, Emil Post and others. This work led to the development of a new subfield of mathematical logic, the *recursion theory*. This area of mathematics is of fundamental importance for computer science and for the theory of computation (where it is called the *computability theory*). But there are also influences and overlaps with such areas of mathematics as proof theory, effective descriptive set theory and even the group theory (through celebrated Higman's embedding theorem).

The question "What does it mean for a function on the natural numbers to be computable?" may be answered in mathematical terms without referring to any computation (essentially by looking at the closure of primitive recursive functions under proper minimalization). Therefore we start our lectures with the development of PRC classes, primitive recursive functions and operations on them. Only then we will introduce a very simple programming language and partially computable functions. After those tools are developed, we continue with coding objects by natural numbers, the universal program, RE sets, oracles and arithmetic hierarchy.

The course is designed to be an introduction to Recursion Theory for incoming graduate students in mathematics, as well as the undergraduate students in mathematics and computer science. In order to understand the main topics in the course, it suffices that the students have some experience with proof-oriented mathematics. All advanced materials will be developed in the course when needed. There will be a strong emphasis on reading and understanding mathematical arguments and on mathematical writing component.

Anticipated Audience/Demand:

The course is appropriate for students interested in pure mathematics and/or in mathematical foundations of computer science. The anticipated audience are graduate and undergraduate students in mathematics and computer sciences who want to learn the basics of the Recursion Theory.

Major Topics:

1) Review of the notation and preliminary concepts (set operations, predicates, quantifiers).

2) Primitive recursive functions and primitive recursively closed (PRC) classes of functions.

3) The closure properties of PRC classes.

4) A language of programming which will be used to model computations. Programs written in this language.

5) Computable functions.

6) Programms coded by integers, universal program and not computable predicates.

7) Recursively Enumerable (RE) sets.

8) Reducibility, completeness, Rice's theorem, the Recursion theorem.

9) Computable (aka recursive) vs primitive recursive, more characterizations of both classes and examples of recursive but not primitive recursive functions.

10) Oracles and relativization to an oracle.

11) The arithmetic hierarchy, Kleene's theorem and Post's theorem.

Methods:

This course will be presented by lecture and class discussions.

Student Role:

Students must attend and participate in class in addition to completing course requirements. Graded work will include homework problems, two or three midterm exams, and a comprehensive final exam.

Graduate students are also expected to do additional reading and assignments. Those would be assigned on an individual base taking into account the background and interests of the students.

Textbook:

The course will be based on lecture notes.

- ***Introduction to the Theory of Recursive Functions*** by Andrzej Roslanowski (the notes will be freely available to the students from the course website).

The graduate students may also have reading assignments from:

Handbook of mathematical logic. Part C. Recursion theory. Edited by Jon Barwise. With the cooperation of H. J. Keisler, K. Kunen, Y. N. Moschovakis and A. S. Troelstra. Studies in Logic and the Foundations of Mathematics, Vol. 90. North-Holland Publishing Co., Amsterdam-New York-Oxford, 1977.

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