Solution to Problems ♢–7

**Problem A:** Can you divide the three dimensional Euclidean space $\mathbb{R}^3$ into 2017 congruent disjoint pieces?

**Answer:** Yes. For $i = 0, 1, 2, 3, \ldots, 2016$ let

$$A_i = \left\{(x, y, z) \in \mathbb{R}^3 : (\exists k \in \mathbb{Z})(2017k + i \leq x < 2017k + i + 1)\right\}.$$

It should be clear that $\mathbb{R}^3 = \bigcup_{i=0}^{2016} A_i$, $A_i \cap A_j = \emptyset$ for distinct $i, j$ and if $i < j$ then the translation by the vector $(j - i, 0, 0)$ maps the set $A_i$ onto the set $A_j$.

Correct solution was received from:

(1) Cody Anderson

POW 7A: ♢
Problem B: Does there exist a subset $X$ of the plane with the property that the orthogonal projection of $X$ onto any line is the union of two disjoint open line segments?

Answer: Yes, a set $X$ with the required property can be obtained as the union of three open discs of radius 1 centered at the vertices of an equilateral triangle with side of length 4 and the set consisting of three points on the edges of the discs. [See the picture below; the arrows point to the three boundary points being added to the union of the three discs.]