MATHEMATICS
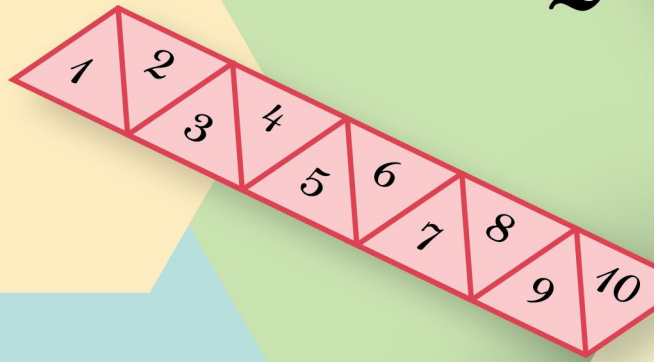
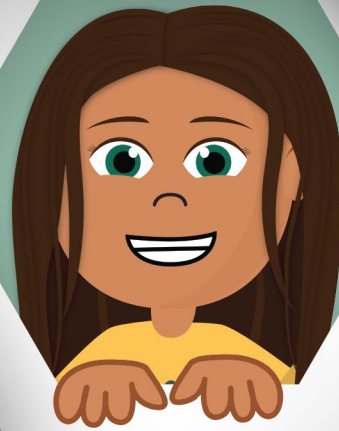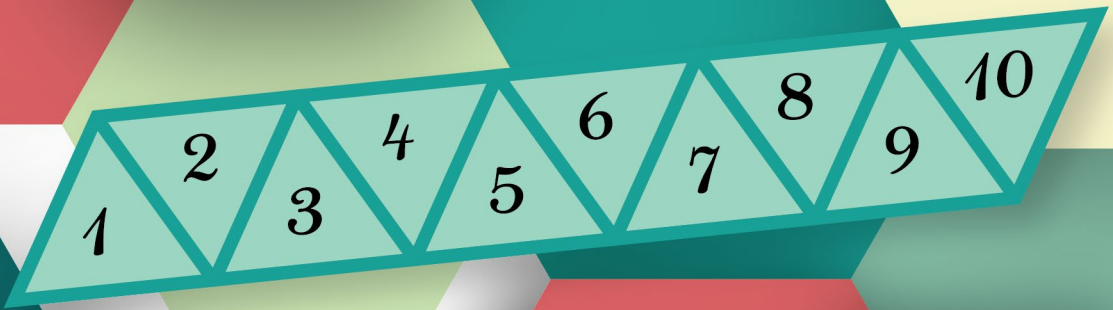## Hexaflexagons

Objective

The student will understand the significance of shapes and how they can affect our understanding of the universe.

The student will be able to build a hexaflexagon.

Vocabulary

Hexaflexagon: A folded paper construction that can be flexed along its folds to reveal and conceal its faces alternately.

Geometry: The area of mathematics that deals with points, lines, shapes, and space.

Equilateral Triangle: A triangle with three equal sides and three equal angles.

Mobius Strip: A two dimensional, non-orientable object that has only one side and one edge.

Hexagon: A six-sided geometric shape made up of triangles.

Background

What is a flexagon, you ask? At first glance it looks innocuous enough, like a folded hexagon or square, a child's fortune teller or cootie catcher, or a piece of origami. But look closely and you will see hidden layers lurking between the front and back. When you fold or pinch corners together, the flexagon "flexes," meaning a formerly hidden layer will come to light as the top layer folds underneath. It all sounds complicated but is really pretty simple when you see an actual flexagon in action.

Mathematicians refer to flexagons as "mathematical oddities." That's because flexagons have very complex mathematical structures. As the flexagon is flexed, sections shift position to create an almost kaleidoscopic effect, and different faces come into view, in cyclic order. Mathematicians enjoy analyzing the structure and dynamic behavior of flexagons. Laypeople just enjoy playing with them.

Materials

- Markers
- Pens
- Scissors
- Hexaflexagon handouts
- Tape/Glue
- iPad or computer

Procedure

1. First, ask students to define a Möbius strip and share whether they believe that a Möbius strip only has one side and one edge.
2. Have them build a Möbius strip and test it by tracing the edge and side.
3. Watch the video:

https://www.youtube.com/watch?v=VIVIegSt8I)

4. Hand out the hexaflexagon sheets to each student.
5. As the students follow along, use your handout to explain how to build a hexaflexagon.
6. Allow students to color in the sides of the surface.
7. Discuss vocabulary terms as the students are working.
8. If time permits, show this video:

https://www.khanacademy.org/math/math-for-fun-and-glory/vi-hart/hexaflexagons/v/flex-mex

9. Optional Extension Activity: Challenge students to find another way to fold a hexaflexagon and/or make another flexagon shape.

Guiding Questions

How many sides does a hexaflexagon have?

How about a hexagon?

How many sides and edges does a Möbius strip have and why?

Career/Future Application

Spatial recognition is a valuable skill in many careers. Engineers incorporate shapes and an understanding of their functions into their everyday job.

Sources

http://www.wikihow.com/Fold-a-Hexaflexagon

http://mathequalslove.blogspot.com/2012/12/hexaflexagon-love.html

http://www.puzzles.com/hexaflexagon/img/instructions_for_folding_trihexaflexagon.pdf

http://www.puzzles.com/hexaflexagon/img/blank_trihexaflexagon_template.pdf

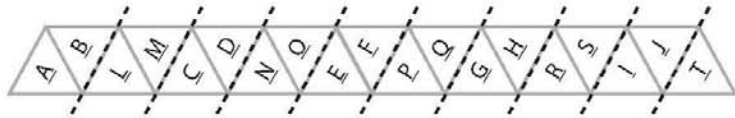https://www.khanacademy.org/math/math-for-fun-and-glory/vi-hart/hexaflexagons/v/flex-mex

HEXAHEXAFLEXAGON BLANK TEMPLATE

INSTRUCTIONS

1. CUT OUT THE TEMPLATE & LIGHTLY LABEL IT WITH PENCIL AS SHOWN BELOW. LATER, YOU CAN ERASE THE PENCIL AND DECORATE YOUR FLEXAGON.
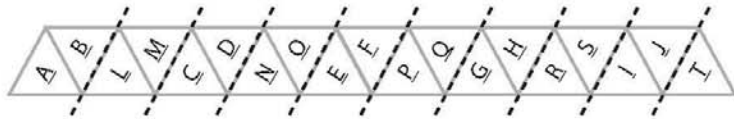
FRONT

A B L M C D N O E E P Q G H R S I I T

BACK

K

2. MAKE MOUNTAIN FOLDS ALONG THE LINES SHOWN BELOW:

FOLDS

A B L M C D N O E E P Q G H R S I I T

3. NOW FOLD THE STRIP SO THAT IT LOOKS LIKE THE DIAGRAM BELOW:
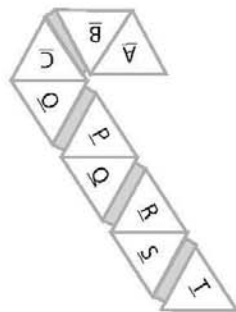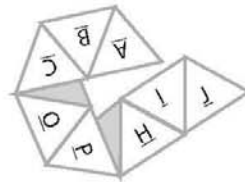
FRONT

A C E G I
B D F H J
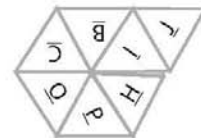
BACK

L N P R T
K M O Q S

4. NOW FOLD YOUR STRIP IN TO THE SHAPE SHOWN HERE:

5. ...AND FOLD AGAIN LIKE SO:

6. BEND THE J FLAP AWAY FROM YOU & GLUE FACE T TO FACE K.

..AND NOW YOU HAVE A WORKING HEXAHEXAFLEXAGON!