

Project Title: Solution of Eigenvalue problem for the confinement of vibrations in non homogeneous beams

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Description: Flexible structures, such as aerospace and ship structures, large communication antennas, and seismically excited buildings and bridges, are exposed to various external and/or internal excitations. They often excite unwanted structural resonances, which are likely to damage or transmit vibrational energy to distant parts or regions where they cannot be tolerated. Therefore, it may be of interest to remove the vibrational energy from the more sensitive parts of the structure, such as the end-effector of a flexible robot manipulator, and transfer it to the less sensitive parts. The sensitive parts of a flexible structure are the set of spatial regions in which vibrations must be eliminated to ensure a better structural performance and less chances of damage. The confinement success requires that the modes shapes of the structure be modified. This modification results in a new class of mode shapes. Mathematically, the vibration confinement can be defined as reducing the absolute value of the amplitude $U(x)$ in the sensitive parts characterized by their spatial domains $[x_1^i, x_2^i]$ ($i=1, 2, \dots, n$). This means that the vibration amplitudes associated with the sensitive elements are lower than those in the remaining parts of the structure.

The concept of vibration confinement is of great importance in several areas, such as mechanical and civil engineering structures. In this project, we consider the confinement of vibrations in non-homogeneous beams in which both physical and geometrical properties are assumed to be functions of the space. The solution of the confinement problem is numerically studied using higher order methods.

Problem Formulation and Objective: Consider the flexural vibration of a beam with variable physical and geometrical properties. The dynamics of its free vibration is governed by the following Euler-Bernoulli equation and associated boundary conditions:

$$\frac{\partial^2}{\partial x^2} \left[E(x)I(x) \frac{\partial^2 u(x,t)}{\partial x^2} \right] + \rho(x)A(x) \frac{\partial^2 u(x,t)}{\partial t^2} = 0, \quad 0 < x < L$$

$$u(0, t) = u(L, t) = 0, \quad \frac{\partial^\varepsilon u}{\partial x^\varepsilon}(0, t) = \frac{\partial^\varepsilon u}{\partial x^\varepsilon}(L, t) = 0$$

where $\varepsilon = 1$ for a fixed-fixed beam, $\varepsilon = 2$ for a simply supported beam and $u = u(x, t)$ is the transverse deflection and L is the beam length. Here, we propose spatial variations of the beam

geometrical properties, cross-section area $A(x)$, and second moment of area $I(x)$, and the physical properties, modulus of elasticity $E(x)$ and mass density $\rho(x)$.

The objective of this study is

- To formulate the problem of confinement of flexural vibrations in non-homogeneous beams,
- To solve the Forward Eigenvalue Problem : solves for the Frequencies and Mode Shapes once the Geometrical & Physical Properties of the Beam are given. In other words, we will use a numerical method to find the natural frequencies and the mode shapes for any set of beam parameters.
- To solve the Inverse Eigenvalue Problem: solves for the Geometrical & Physical Properties of the Beam once the Frequencies and Mode Shapes are given. A numerical method will be employed to discretize the beam spatial domain for the purpose of approximating the Geometrical & Physical Properties of the Beam.

Prerequisites: Math 2350 is required. Math 3300 recommended, but not required. You will be using one of the following programming language (Matlab, Mathematica, Maple, C or C++)