Project Title: Adaptive Methods for hyperbolic partial differential equations.

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Description: The discontinuous Galerkin (DG) method is a class of finite element methods which use completely discontinuous functions as the numerical solutions and test functions. DG method has been under rapid development in recent years as one of the powerful numerical methods to solve various partial differential equations (PDEs), especially convection dominated PDEs in science and engineering. Stable and convergent discontinuous Galerkin methods have been designed in recent years for various types of PDEs, including hyperbolic, parabolic and elliptic equations. DG methods have found rapid applications recently, in such diverse areas as aeroacoustics, electromagnetism, gas dynamics, granular flows, meteorology, oceanography, semiconductor device simulation, turbulent flow, weather forecasting, among many others.

The DG method is an appealing approach to address problems having discontinuities, such as those that arise in hyperbolic conservation laws. The DG methods are a family of locally conservative, stable and high-order accurate methods that are easily coupled with other well-known methods and that are well-suited to adaptive strategies. For these reasons, they have attracted the attention of many researchers working in computational fluid dynamics, computational mechanics, computational mathematics and computer science. The main advantages of the DG methods include its flexibility in triangulations including non-conforming meshes, its easiness for hp-adaptivity, its robustness in incorporation of shock capturing techniques such as limiters, and its high parallel efficiency.

We propose to study the superconvergence properties of DG solutions of hyperbolic problems. We will perform a numerical and theoretical study of the existence of superconvergent points for DG methods applied to one dimension hyperbolic problems. We will present several numerical results confirming the superconvergence rates.

In order for the DGM to be useful in an adaptive setting, techniques for estimating the discretization errors should be available both to guide adaptive enrichment and to provide a stopping criteria for the solution process. We will use the superconvergence results to construct simple, efficient, and asymptotically correct a posteriori error estimates by solving local hyperbolic problems with no boundary conditions. The error estimation procedure does not require communication across neighboring elements which makes it useful for parallel computations. A posteriori error estimates are needed to guide adaptive enrichment and to provide a measure of solution accuracy for any numerical method.

Finally we propose a new adaptive method for scalar conservation laws. An a posteriori error estimate will be used to design the adaptive strategy that automatically refined the mesh during the simulation. The adaptive method will be tested by numerical results with both continuous and discontinuous solutions that validate our theory.

Prerequisites: Math 4330/8330 is required. You will be using the software Matlab.

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