University of Nebraska at Omaha Department of Mathematics

Fall 2002 Newsletter

November 2002 Volume 4, Issue 1

Earl Scholarship Fund Endowed

Last Spring an anonymous donor contributed $125,000 to the James Earl Scholarship Fund. Starting this academic year, 2002-2003, $5,000 will be divided each year between several outstanding undergraduate mathematics majors. This year the awardees are Aaron Becker, Andrew Gacek, Eric Manley and Zachary Zaiss.

Aaron Becker is a joint CS/Math/Philosophy/Political Science major and Walter Scott Scholar from Randolph, NE. His favorite mathematics course at UNO is Number Theory. His future plans include going to lawschool. "Thinking abstractly is a pleasurable cognitive experience."

Andrew Gacek is a joint CS/Math major and Walter Scott Scholar from Robbinsdale, MN. His favorite course here is Abstract Algebra. As a sophomore, Andrew placed 78th nationally on the very challenging Putnam Exam. "Finding a solution (to a math problem) has to be one of the greatest feelings in the world."

Eric Manley attended Gross High School in Omaha. He is double majoring in CS/Math at UNO. His favorite math course is Number Theory/Cryptography. In Fall 2002 he is helping the Mathematics Department launch a high school Problem of the Week competition. His career plans are uncertain but will involve mathematics.

Zachary Zaiss attended Millard South High School in Omaha. He is a Walter Scott Scholar majoring in CS/Math/Psychology and minoring in Spanish. His favorite math course is Number Theory/Cryptography. He too is helping the Mathematics Department start the high school Problem of the Week this fall. He likes mathematics because it gives him a deeper understanding of computer science.

Earl Scholarship Awardees: *Andrew Gacek, Aaron Becker, Zach Zaiss, Eric Manley.*

Pictured Above: *Jim and Sylvia Earl, Jr., recent visitors to the Mathematics Department.*

From The Chair

Interest in Mathematics at UNO is growing rapidly. During academic year 2001-2002 Student Credit Hour production was 17,500 giving our department the largest enrollments on campus (History is second at 16,500 SCH, English and Communications next). And our classes are fuller than ever this fall. The Peter Kiewit Institute, which houses UNO's popular technology programs, is a major, but not the only reason. Lots of people besides just computer scientists and engineers are realizing that an understanding of mathematics is necessary for advanced use of computers.

We have two new faculty this year, Mr. Jerome Drakeford, a specialist in mathematics education and also Dr. Jim Rogers in a research position. Read about their pursuits in this issue.

As you know already UNO is attracting some of the finest undergraduate students in the region. We are now able to recognize their academic achievements more visibly with the newly endowed Earl Scholarship Fund. You could almost say that Dr. James Earl is the history of our department, having served as Professor and Chair from 1932 – 1962. Upon Dr. Earl's death in 1972, his wife Margaret established a scholarship fund in his name. Now that it has been endowed, it is expected to provide about $5000 per year to be divided among deserving undergraduate math majors for many years to come.

The department has a talented and hard working faculty and student body. These annual newsletters, begun by Dr. Margaret Gessaman in 1999, attempt to highlight the departmental activities.

UNO alumni should be proud of their Alma Mater!

Jack Heidel, Chair

Dr. Jack Heidel with Griff Fryer'44

Mathematicians have tried in vain to this day to discover some order in the sequence of prime numbers, and we have reason to believe that it is a mystery into which the human mind will never penetrate.
~ Leonhard Euler (1707-1783)

Inside this issue:

News of Recent Graduates

December 2001

Martha Lynn Blue (BA)
Paul Anton Holm (BA), *Seeking an actuarial position in Omaha.*
Sharon Jones (MAT), *Part-time instructor at UNO Mathematics.*
Mandy Irene Klockner (BS), *Graduate Assistant at UNO Mathematics.*
Jennifer Yvonne Waldren (BA), *Full-time dancer with the Rose Theatre in Omaha. She will most likely seek a master's degree in actuarial science.*

August 2002

Shailendra Basnet (BS)
Elisa Lowin Booth (BGS), *Teaching Math at Beatrice High School and is working to complete her education accreditation for teaching.*
Amanda Jane (Lang) Kunes (BS)
Rachel Ann Neurath (MAT)
Carl Scheurmann (MS), *Currently looking for a teaching position in mathematics in the Omaha area.*
Richard Schultz (BS), *Working at the Douglas County Health Department in Omaha as a data analyst. He is also studying Geographic Information Systems.*

May 2002

Anthony Armstrong (BS)
Matt Culek (BS), *Graduate student in mathematics at the University of Texas, Austin.*
Chris Farrow (MA), *Ph.D. student in physics at Michigan State.*
Nick Hildenbrandt (BS), *Graduate student in computer science at the University of Illinois, Urbana-Champaign.*
Dennis Johnson (BGS), *Commissioned into the Air Force.*
Kristin King (BS)
Jim Rogers (MS), *Visiting Assistant Professor in the Mathematics Department at UNO.*
Crystal Stumme (BS)
Shelly Sutherland (BGS), *Employed by Physician's Mutual Insurance Co. as an actuarial assistant.*

GTA Andrew Buchan hard at work in the Walk-in Tutor Room

The University of Nebraska Foundation

Mary Bernier

What is the University of Nebraska Foundation?

Established in 1936, the University of Nebraska Foundation exists to raise and manage private gifts to further educational excellence on all four campuses of the University of Nebraska. The Foundation is a private, nonprofit corporation designated by the University of Nebraska Board of Regents as the primary fund-raiser and manager of gifts to the University from alumni, friends, corporations and other foundations. Though it is separate from the University, the Foundation works closely with the University to generate private gifts for the University's major fundraising priorities. The University of Nebraska Foundation exists solely for the benefit of the University of Nebraska. If you would like more information about the NU Foundation or making a gift for UNO, please contact Mary Macchietto Bernier, Director of Development, in the Omaha Office, 402-595-2302. mbernier@nufnd.uneb.edu

Readers Respond

God does not care about our mathematical difficulties, he integrates empirically.
~ Albert Einstein (1879-1955)

Jim Earl's memories of UNO Mathematics in the early days: Harry Rice was my father's sidekick for as long as I can remember, but I don't remember much about him, except that he lived in Council Bluffs. Dr. Hunziger came to Omaha after I left in 1949. I don't believe I ever met him and don't remember much that my father said about him. Bill Walden was the head of what was then the computer center and was a moving force behind the development of the computer science major. I remember Dr. Haeder. During my father's final illness, he developed severe problems at a conference, and Dr. Haeder helped him get back to Omaha. Sylvia and I really enjoyed our visit to your department and were very impressed how far it has come since the days of my father. We were delighted to hear from Jim Leslie about the recent major gift to replenish the mathematics scholarship. jearl@astro.umd.edu

Griff Fryer, (BA, 1944): Wataga, IL writes that he would like to develop and publish a book on virtual coordinate systems drawing on his long experience as a mathematician for Boeing Company. The Mathematics Department is currently working with him to accomplish this.
grusfryrjr@webtv.net

Captain David Petersen (MS, 1956): I have a varied academic background. I was acting Chairman of the Physics Department at Dana College in the 1960's. My B.G.E. is from UNO, my M.S.E.E. is from the University of Nebraska at Lincoln, my J.D. is from Creighton University School of Law and my LL.M. is from Emory University School of Law. The four texts I have out include one in medical engineering, two in constitutional law and one in courtroom procedure. The titles are: Waveform Analysis in Medicine; Pre-Trial Criminal Procedure; A Field Manual of Criminal Law of Police Procedure; and The Police Officer in Court. Recently, I completed several book proposals in a variety of fields. All related to my Ph.D. work. My Ph.D. will be in Conflict Management from Trinity College/University of Liverpool. captainrdp@att.net

Chad Fulk (MA, 2001): Ph.D. student at UIC in high energy physics.
cfulk1@uic.edu

George Phillips Jr. (BGS, 1970) writes: Went on to the University of Missouri Rolla for a Masters in Engineering Mgt in 1974. Worked for McDonnell Douglas from 1970-1974 using mathematical models to design the air inlet on the F-15 fighter. I finally found a use for that Projective Geometry class. Went on to Southwestern Bell Tele Co, 1975-2000 where I worked in the IT and Network organizations implementing a number of Major Operations Support computer systems. Had to apply the logic here. I retired from SBC in 2000 and took a week off and am now working for Science Applications International Corporation/Telcordia designing operations support computer systems for the Telecom Industry. From 1974-1998 I was a part-time Instructor in Mathematics and Computer Science at St. Charles Country (Missouri) Community College. Had to give it up due to a business relocation, but really enjoyed staying in touch with the academics. I also taught a couple of semesters at Webster University here in St. Louis. I really admired the part-time instructors at UNO, and the way they brought real life situations to the class room. I really enjoyed my time at UNO and can't tell you how much the education UNO provided has meant to me and my family. Tell everyone thanks and keep on doing the great job that they have done for so many years. It was appreciated.
gphillip@telecordia.com

Richard Schultz (BS, 2002): writes about getting a job as a data analyst at the Douglas County Health Department: Without a doubt, the two projects we did in Stats II were instrumental in getting the job. They made photocopies of my work on both the group project I did with Gary and Anthony, and the solo one I did on the political campaign logos. They also grilled me hard about them and made me explain what I was proposing mathematically, and defend it. They were looking for someone who could interact and answer questions verbally, and who could explain statistical concepts in everyday language. I didn't feel like I did all that well at the time, but they must have been impressed enough!
sawdust@novia.net

The Department of Mathematics 2002-2003 Faculty and Staff

Mary Dennison, *Assistant Director-Math Lab*
Parsla Dineen, *Instructor*
Judith Downey, *Instructor*
J. Scott Downing, *Professor*
Jerome Drakeford, *Instructor*
G. Griff Elder, *Associate Professor*
Steve From, *Associate Professor*
Margaret Gessaman, *Professor Emeritus*
Jack Heidel, *Chair and Professor*
Betty Hickman, *Associate Professor*
John Konvalina, *Professor*
Yi-Hsin Liu, *Professor*
Margaret Mainelli, *Staff Assistant*
John Maloney, *Professor*
Dora Matache, *Assistant Professor*
Valentin Matache, *Assistant Professor*
Janice Rech, *Associate Professor, Director-Math Lab*
Jim Rogers, *Visiting Assistant Professor*
Andrzej Roslanowski, *Assistant Professor*
Dean Ann Saenz, *Special Projects*
Laura Schaben, *Instructor*
Larry Stephens, *Professor*
Cindy Teller, *Staff Assistant-Math Lab*
Yanxing Song, *Research Associate*
Zhenyuan Wang, *Associate Professor*

Jerome Drakeford *New Faculty Member*

Jerome Drakeford

Mr. Jerome Drakeford (MS, Physics, Creighton University, 1995) joined the Mathematics Department in Fall 2002 as an Instructor. Jerome is an expert at teaching MATH 1310, Intermediate Algebra, to the "mathematically challenged." Since 1994 Jerome served as the math/science tutor for UNO's Project Achieve, a federally funded program to aid disadvantaged students. During these years, he also taught special low enrollment sections of Intermediate Algebra for UNO's Goodrich Program.

Thus, Jerome is uniquely qualified to help the department revamp its campus-wide mathematics requirement, MATH 1310, which is so frustrating to many students. Besides offering low enrollment sections to give more attention to students with math anxiety, the department is also considering content changes to put greater emphasis on general quantitative literacy rather than just traditional algebraic symbol manipulation.

Jerome has extensive experience in teaching large lecture classes, special limited enrollment classes and one-on-one tutoring at several Omaha area colleges. He has also tutored for the Omaha Public Schools and worked in industry. The department is very happy to have Jerome's help as it tries to address one of the thorniest issues in mathematics education today. How can all students be motivated/cajoled into learning enough basic math in order to thrive in an increasingly technological society and world.

Mary Zeleny

Jerome is trying out several innovative techniques to accomplish these goals. First of all, his sections meet four hours a week for three hour credit, with mandatory attendance. Outside tutoring is also available. In addition, Jerome is employing the services of a professional counselor, Ms. Mary Zeleny. Mary had to struggle through MATH 1310 herself as a student. She is an expert at drawing students out, getting them to articulate their anxiety, both verbally and in writing. (Sample: $(5m-1)(5m+1)=25m^2 + 5m-5m-1=25m^2 -1$, "The steps are definitely more complicated. I need to follow the F.O.I.L. method closer. However, so far I think I understand <u>everything</u>"). Mary meets briefly with each class at least once a week, addressing such issues as studying styles, math motivation and gender/cultural issues.

POW Explodes Into New Areas

The POW (Problem of the Week) Contest has continued to generate student interest and participation. In the Fall semester 2001 about 20 students submitted one or more correct solutions. The winners for the Fall semester were Andrew Gacek (1st place), Jared Davis (2nd place) and Hing Lim Chan (3rd place). Pictures of these "Masters" can be seen on the math department web-site: www.unomaha.edu/~wwwmath/

In order to encourage more student participation, the POW Math Faculty Committee decided to expand the contest to two levels for the Spring semester 2002. A "Rockies" level with problems that are accessible to all undergraduate students and a "Himalayas" level with problems aimed at upper classmen and graduate students was started. Over thirty students submitted one or more correct solutions in the two levels in the Spring semester 2002. The successful "scalers of the Himalayas" were: Andrew Gacek (1st place), Hing Lim Chan (2nd place) and Eric Manley (3rd place). The successful "scalers of the Rockies" were Mark Hanus (1st place) and there was a tie for 2nd and 3rd place between Ashley Sedlacek and Josh Weatherley.

The contest is going strong in the Fall semester 2002 at the two levels. Faculty and friends submitting and grading problems for the contest include: Charlie Downey, Judy Downey, Scott Downing, Griff Elder, John Konvalina, Dora Matache and Valentin Matache. Examples of problems and solutions can be viewed on the math department web-site. In connection with Math Awareness Month in April, 2002 the mathematics department at UNO sponsored a POW contest for area high school students. This was an on-line contest throughout the month of April. Two problems were posted on the math department web-site on Monday morning of each week in April. The students were given until Friday of each week to submit their solutions by email, fax or regular mail. Over 80 high school students from 8 different area schools submitted one or more correct solutions. Students submitting correct solutions to all 8 problems were: Robert Crider of Bellevue East High School, Jenny Kane of Papillion-La Vista High School, Igor Konfisakhar of Westside High School Jason Olson of Bellevue East High School and Chris Villa of Westside High School. Faculty involved in this contest included Parsla Dineen, Judy Downey, Dora Matache and Valentin Matache.

The UNO Math Club decided to continue the on-line POW Contest for area high school students throughout the school year beginning in the Fall semester 2002. They have had over seventy five students from many different schools participate in the contest so far. Three undergraduate students, Josh Hubbard, Eric Manley and Zach Zaiss have designed the web-site and have been responsible for all work associated with this contest. Plans are for this contest to culminate in an on-site competition among the top scorers throughout the year during Math Awareness Month in April 2003 at UNO with prizes being awarded to the winners. The web-site for this contest is: www.unomaha.edu/~pow.

UNO Math Club

New officers were elected for the UNO Math Club in the Spring 2002. They are: Steve Bills, President; Eric Manley, Vice-President; Josh Hubbard, Treasurer; and Adrick Magonga, Secretary. Meetings this Fall have included a math movie night at Scott Hall in September and a talk on "Cryptography, Ciphers and Mathematics" by Griff Elder and John Clark in October. Future plans include a math game night in November. The Math Club participated in the Activities Fair at UNO in August where several new members joined. Pictured at the Math Club booth at the Activities Fair is Math Club President, Steve Bills. Faculty sponsors for the 2002-2003 school year are Parsla Dineen and Judy Downey.

Putnam, Putnam, PUTNAM

Riding the broad shoulders of Andrew Gacek, Hing Lim Chan and Eric Manley, UNO placed 97/453 last December on the 62nd Annual William Lowell Putnam Mathematical Competition. Andrew led the hearty band of seven participants with 51 points (placing 78/2954 in the US and Canada). Hing earned 14 points (placing in the top 20%) while Eric scored 10 points (placing in the top third). Altogether this gave UNO its strongest showing ever!

Here are two of the problems they handled successfully:

A-3. For each integer m, consider the polynomial $P_m(x) = x^4 - (2m+4)x^2+(m-2)^2$

For what values of m is $P_m(x)$ the product of two non-constant polynomials with integer coefficients?

B-1. Let n be an even positive integer. Write the numbers 1, 2,….n^2 in the squares of an $n \times n$ grid so that the k-th row, from left to right, is $(k-1)n + 1, (k-1)n + 2, \ldots\ldots, (k-1)n + n$

Color the squares of the grid so that half of the squares in each row and in each column are red and the other half are black (a checkerboard coloring is one possibility). Prove that for each coloring, the sum of the numbers on the red squares is equal to the sum of the numbers on the black squares.

Excerpts from Mathematics Program Review

(Held during February, 2002)

Faculty and Staff
The department deserves high marks for scholarship. Many faculty engage in significant ongoing publishing and professional activities. Several faculty members have research records that one would expect from departments with doctoral programs…

Curriculum
The UNO Mathematics Department offers a solid curriculum that follows the guidelines established by the Mathematical Association of America (MAA). Recently, the department has wisely expanded its offering in statistics. The demand for people with training in statistics is so great that even more courses in this area should be offered…

Student Comments
The department arranged for the committee to meet with several PKI students who are taking classes in the Mathematics Department. They described the faculty as dedicated and passionate about teaching. The students were very pleased with the accessibility and friendliness of the faculty. The students said that the Mathematics faculty were more accessible than those from other departments. One student said: "They go out of their way to make it known that you are wanted in the Mathematics Department"…

Outreach Efforts:
The committee is pleased with the department's participation in the Putnam Competition and the existence of the "Problem of the Week" contest…

Committee Concerns
The committee is concerned whether the department has sufficient resources to provide the Scott Scholars with a challenging curriculum.

Over the past decade there has been considerable interest in many mathematics departments to involve undergraduate students in meaningful research. In light of the recent hires in the department who have strong research credentials and great passion for working with PKI students, the UNO Mathematics Department has outstanding potential to excel in this area…

Yet another excellent way to provide opportunities for the highly talented students at UNO is to offer a summer research program modeled after the NSF sponsored Research Experience for Undergraduates (REU) programs…

Specific Recommendations
Although some new faculty positions have been recently granted, the UNO Mathematics faculty is still stretched to the limit. More tenure track positions are important for the well being of the mathematics program and the PKI program. Certainly most of the initiatives we recommend in this report cannot be done without more faculty…

We propose that the department seek donors to establish cash prizes for a variety of student achievements. Among the possibilities are awards for performance in the Putnam Competition, outstanding graduating senior, outstanding service to the department, and an Outstanding Freshman Mathematics Prize. At many schools, retiring faculty request that money be given in their name for prizes rather than contributing to a retirement gift…

We used to think that if we knew one, we knew two, because one and one are two. We are finding that we must learn a great deal more about 'and'. *~ Arthur Eddington (1882-1944)*

National Science Foundation (NSF) Grant

The National Science Foundation recently awarded Dr. Griff Elder a three-year, $117,224 grant in support of his research in the field of Algebraic Number Theory. This field begins simply enough, with the study of basic multiplicative properties of the whole numbers. It is currently 'hot' as a result of applications in cryptography and coding theory. But efforts, long ago, to resolve such long-standing problems as Fermat's Last Theorem led mathematicians to expand their horizons to include algebraic numbers (such as $\sqrt{-3}$). Griff's research focuses on the fundamental properties of these expanded sets. In particular, using symmetry, he investigates the internal structure of basic subsets: additive ones (generalizing the integers) and multiplicative ones (generalizing {+1, -1}). The title of the grant proposal was "Galois Structures in Local Number Fields". Beyond supporting his own research, this award provides support for a small number of undergraduate researchers who will work intensively with Griff on some open problems in Algebraic Number Theory. Their goal (and his) will be to produce publishable results.

National Institute of Health (NIH) Grant

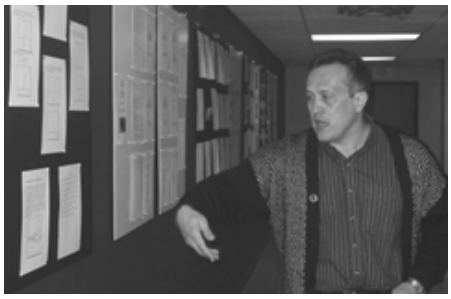
The Mathematical Cell Biology Research Group in the Department of Mathematics has been awarded a three-year, $450,000 grant by the National Institute of Health (NIH) for the project *Attractors of Complex Signal Transduction Systems.* The topic of study is using mathematical analysis to understand the structure and function of complex biochemical pathways. It is a true interdisciplinary research project that involves state of the art research in both mathematics and cell biology. The grant includes funding for both postdoctoral fellows and graduate research assistants for the department.

Several research funding agencies, including NSF and NIH, are convinced that a new era in biology is dawning in which research in biological systems will be dependent on advanced mathematics. This is because biomedical researchers are uncovering vast layers of complexity in living systems that cannot be studied with standard laboratory techniques. Thus, an effort is being made to recruit mathematicians to work on biological problems and support their research with dedicated funds. The ultimate goal of the NIH is to make mathematics in the future as integral to biology as it currently is to the physical sciences.

The Mathematical Cell Biology Research Group in the Department of Mathematics at UNOmaha fits this new goal perfectly. Created as a result of a long-term collaboration between Dr. Jack Heidel and Dr. Jim Rogers, a former researcher at the University of Nebraska Medical Center, the group is dedicated to applying mathematical analysis to biochemical systems. Specifically, they will be creating a new Boolean model of three large, interconnected biochemical networks and then developing new mathematical tools for the exact analysis of the Boolean model. They will be looking for chaos and mathematical complexity in the model in order to determine how these phenomena might be involved in information processing in the living cell. The project falls into the overall category of cancer research as malfunctions in the biochemical networks involved in the study are commonly associated with cancer.

A postdoc is expected to be hired by the end of the year, and a new educational program in mathematical cell biology has been created. Students working with the Mathematical Cell Biology Research Group will have the opportunity to get a Master's degree in mathematics and a Ph.D. in cell biology. In addition, Dr. Rogers has joined the mathematics department as a Visiting Assistant Professor.

An Invitation to Mathematical Analysis



Calculus is, (at a very basic and predominantly computational level) the ABC of what is called nowadays Mathematical Analysis. Its importance for mathematics and science can be understood by observing the presence of Calculus courses in any higher level educational program in science or engineering, world-wide. Analysis is a scholastic term, consisting of the Greek words: ana (after) and lysis (cut). Everyone who understood the construction of the Riemann integral, (by "asymptotic partitioning"), will get the message. Dismantle, for scientific purposes, the whole in its components, then study them to understand the system. That, would be, in a nutshell the definition of the analytic method.

Freshman-calculus is basic, computational, real analysis. The term "real" means that functions over the field of real numbers are studied from an analytic point of view. Some basic concepts of this theory go back to Fermat, but a lot of credit goes to Isaac Newton, whose major contribution was to use calculus to understand the physical phenomena of the surrounding world. Despite the famous dispute for paternity between Newton and Lebniz, integral calculus was incompletely understood until the basic contributions of Cauchy and Riemann. Therefore we teach Riemann integration rather than Leibniz or Newton integration today. Riemann's theory leaves out a lot of important classes of functions, which cannot be integrated. The need of a broader theory was very acute at the turn of the 20-th century. The French mathematician Lebesgue proposed what is called today "the abstract Lebesgue integral in the sense of measure theory". This kind of theory is the peak of any basic course of real analysis. The idea of measure goes back to elementary geometry, another scholastic term consisting of two Greek words: geo (Earth) and metron (to measure). Indeed, the linear Lebesgue measure can be understood as "the idea of length", the planar Lebesgue measure means "area", while the spatial Lebesgue measure means "volume". The Lebesgue integration theory considers in a unitary way a concept of measure on a space of choice and constructs an associated integration theory, (or integral calculus). If the space of choice is the real line endowed with the length-measure, then Riemann's theory becomes a particular case of Lebsesgue's theory, that is Riemann integrable functions are also Lebesgue integrable and the two integrals coincide. The paragraph above is meant to give the unadvised reader the flavor of real analysis. The current one will consider complex analysis. According to the German mathematician Leopold Kronecker, "natural numbers, (that is 1,2,3,.....) were created by God, but anything else is created by Man." Thus negative integers are man-created, but everybody understands negative temperatures, so no need to advocate for the importance and applications of that creation. To say nothing that everybody who measures something, understands that a foot needs to be divided into 12 inches hence, rational numbers, i.e. fractions of the integers had to be created too. According to Pythagoras:"The numbers rule the world." Letting aside the mystic respect for numbers of the Pythagoreans, we must quote the ancient Greek again, and recall that they were aware that no partitioning process of a foot into smaller fractions of 1, will take care of expressing exactly the length of the diagonal of a square having side-length 1 foot. The number under scrutiny is the square root of 2, and fortunately or not, it happens to be irrational. Putting rationals and irrationals together one arrives at the construction of real numbers, the environment of choice of classical, real analysis. The second major flavor analysis comes in, is called complex analysis. Why should people invent such abstract things as complex numbers? To solve the equation "x squared plus 1 equals 0", would an algebraist answer. Indeed adding the imaginary unit to the field of real numbers leads to an algebraically closed field. This mere statement makes the object of the so called "fundamental theorem of algebra " (referred to as the Gauss-D'Alembert theorem, in some texts), saying that polynomial equations with complex coefficients always have at least one complex root. It is interesting that the one and only proof of this celebrated theorem a math major gets to see, is in most cases the one based on a complex analysis theorem by the French mathematician Liouville. The main ingredient in Liouville's theorem is an integral formula due to Augustin Louis Cauchy. Cauchy is one of the major contributors to complex analysis. His integral formula, says that the complex functions differentiable inside a disk have at each point, a value uniquely determined by the values assumed on closed contours surrounding that point. Another striking consequence of Cauchy's integral formula, (other than the Liouville proof of the Gauss-D'Alembert theorem) is the fact that functions, complex-differentiable once on a disk, are automatically differentiable infinitely many times on that disk. This feature, and many others like it make complex analysis, very different from real analysis. Besides Cauchy, the other grand contributor to complex analysis is Riemann, whose contributions are more geometric in nature, as opposed to Cauchy's contributions which are predominantly analytic.

Finally, the last major flavor analysis comes in is called functional analysis. Functional analysis studies spaces consisting of functions, more than anything else. A linear functional is the easiest example of such a "function of functions", because the term designates the scalar-valued functions on various spaces. What is the characterization of the linear functionals on a space of continuous functions? Given the basic role played by continuity in analysis, this should be an interesting question. In a couple of seminal papers published around 1910, the Hungarian mathematician F. Riesz answers: that space can be understood as the collection of all measures on the domain of definition of our continuous functions. It is one of the most celebrated results of functional analysis, sometimes called "the Riesz representation theorem".

In most of my research papers I have studied the transformations of function spaces, by composition with a fixed function. This kind of transforms, designated as Composition Operators, are at the interface of function theory, (that is real and complex analysis) and Operator Theory, a branch of functional analysis. They made the object of intense research during the last four decades and demonstrate the beautiful interplay between the various major branches of Mathematical Analysis. Their study leads to a deeper understanding of the behavior of functions under the fundamental operation of composition.

To give an example, I will briefly describe a paper I published in 2001 in the journal Linear Algebra and its Applications. In it, I studied what is called the numerical range of composition operators and my method was to write software in MATLAB, use it to graph numerical ranges of Composition Operators, and once viewing their shape, prove theoretical classification theorems. Operators can be understood as (possibly infinite) matrices. Their action on the unit ball of the space on which they are defined can be understood better by associating to them a certain complex-valued function whose range is called "the numerical range of the given operator". A well-known theorem proved in 1918 by O. Toeplitz and F. Hausdorff says that numerical ranges are convex sets, that is subsets of the plane with the property that the line-segment joining any two points belonging to the given subset is fully contained by that set. Numerical ranges of Composition Operators induced by very simple functions can have a relatively large variety of shapes and their theoretical determination is not exactly easy. Here are some figures illustrating this statement:

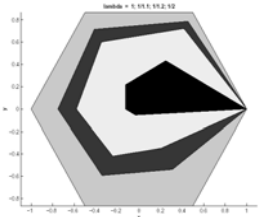


FIGURE 1

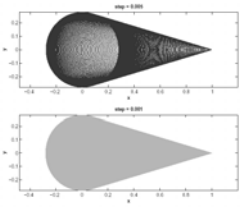


FIGURE 2

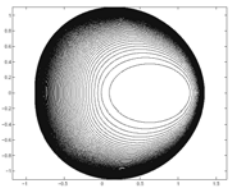


FIGURE 3

Composition Operators can have polygonal numerical ranges. Figure 1 contains several such ranges, nested. Numerical ranges of composition operators can also be "cone-like" (Figure 2), and can have different other shapes. A word on the method used. A computer cannot perform infinitely many operations. Unfortunately, composition operators have infinite matrices. So, what I did was to truncate the infinite matrix to smaller sub-matrices. To understand the action of composition by (z+1)/2, one can consider 2 x 2, then 3 x 3, sub-matrices, and so on. Graphing the boundaries of the numerical ranges of about 200 such truncations gives us a fair approximation of the numerical range of the composition operator induced by the transform (z+1)/2, (Figure 3). More on this subject can be found at http://www.unomaha.edu/vmatache/mathawareness/poster02.html, opening the pdf-file “Numerical Ranges of Matrices”.

This article is mainly addressed to our current and potential students. It is intentionally at a general audience level and should be taken as an invitation to take our analysis sequence.

Valentin Matache

University of Nebraska Omaha

NON-PROFIT ORG.
U. S. POSTAGE PAID
PAID
OMAHA, NE
PERMIT NO. 301

Department of Mathematics
College of Arts and Sciences

6001 Dodge Street
Durham Science Center 203
Omaha, Nebraska 68182-0243

Phone: 402-554-3430
Fax: 402-554-2975
E-mail: margaretmainelli@mail.unomaha.edu
Website: www.unomaha.edu/~wwwmath

We must admit with humility that, while number is purely a product of our minds, space has a reality outside our minds, so that we cannot completely prescribe its properties apriori. ~ *Karl Friedrich Gauss (1777-1855)*

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