

**SOLUTION 1:**

Observe that  $\angle BAF = \angle ADE$  since they are complementary to  $\angle CAD$ . Deduce that triangles  $BAF$  and  $ADE$  are similar. Based on that, write the equality

$$\frac{BF}{AF} = \frac{AE}{DE}$$

Express the above equality as follows

$$\frac{BF}{3 + EF} = \frac{3}{5}. \tag{1}$$

Now observe that  $\angle BCF = \angle CDE$  since they are complementary to  $\angle DCA$ . Thus the triangles  $BCF$  and  $CDE$  are similar. Based on that, write the equality

$$\frac{BF}{CF} = \frac{CE}{DE}$$

Express the above equality as follows

$$\frac{BF}{7 - EF} = \frac{7}{5}. \tag{2}$$

Solve the system consisting of equations (1) and (2), getting  $BF=4.2$ .

**SOLUTION 2:**

Notice that ABCD is a cyclic quadrilateral since opposite angles are supplementary. Extend the segment DE to X on the circumcircle. Note that BFEX is a rectangle, since  $\angle DAB$  subtends the same arc as  $\angle DXB$ . Based on the above observation, deduce that  $BF = EX$ .

Observe that segments AC and DX are intersecting chords with intersection point E and therefore  $DE \times EX = AE \times EC$ . Now write

$$BF = EX = \frac{AE \times EC}{DE} = \frac{21}{5} = 4.2$$

*Any other alternative, correct solution earns full credit.*