

SOLUTION 1:

Observe that $\angle BAF = \angle ADE$ since they are complementary to $\angle CAD$. Deduce that triangles BAF and ADE are similar. Based on that, write the equality

$$\frac{BF}{AF} = \frac{AE}{DE}$$

Express the above equality as follows

$$\frac{BF}{3 + EF} = \frac{3}{5}. \tag{1}$$

Now observe that $\angle BCF = \angle CDE$ since they are complementary to $\angle DCA$. Thus the triangles BCF and CDE are similar. Based on that, write the equality

$$\frac{BF}{CF} = \frac{CE}{DE}$$

Express the above equality as follows

$$\frac{BF}{7 - EF} = \frac{7}{5}. \tag{2}$$

Solve the system consisting of equations (1) and (2), getting $BF=4.2$.

SOLUTION 2:

Notice that ABCD is a cyclic quadrilateral since opposite angles are supplementary. Extend the segment DE to X on the circumcircle. Note that BFEX is a rectangle, since $\angle DAB$ subtends the same arc as $\angle DXB$. Based on the above observation, deduce that $BF = EX$.

Observe that segments AC and DX are intersecting chords with intersection point E and therefore $DE \times EX = AE \times EC$. Now write

$$BF = EX = \frac{AE \times EC}{DE} = \frac{21}{5} = 4.2$$

Any other alternative, correct solution earns full credit.