

MATHEMATICS TEST

SAMPLE QUESTIONS

The following questions illustrate the range of the test in terms of the abilities measured, the disciplines covered, and the difficulty of the questions posed. They should not, however, be considered representative of the entire scope of the test in either content or difficulty. An answer key follows the questions.

1. On a questionnaire, a respondent must choose 3 of the 5 questions presented. How many different combinations of 3 questions can the respondent possibly choose?
 - (A) 10
 - (B) 15
 - (C) 20
 - (D) 30
 - (E) 60

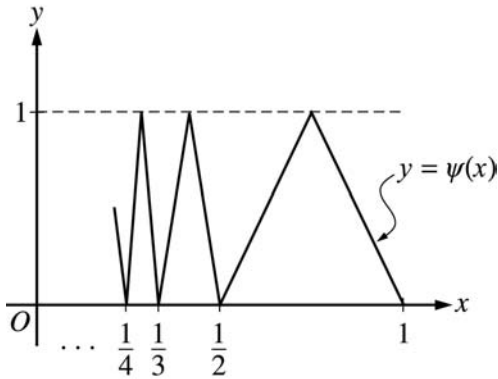
2. A function f has the property that, at every point (x, y) on the curve $y = f(x)$, the slope of the line tangent to the curve is equal to $2xy$. Which of the following best describes the function f ?
 - (A) Linear
 - (B) Trigonometric
 - (C) Inverse trigonometric
 - (D) Logarithmic
 - (E) Exponential

3. Let A and B be metric spaces and let $f : A \rightarrow B$. Suppose that whenever X is an open set in B , the set $\{a \in A : f(a) \in X\}$ is closed in A . Which of the following must be true?
 - I. f is injective
 - II. f is continuous.
 - III. f is a homeomorphism.
 - (A) None
 - (B) II only
 - (C) III only
 - (D) I and III only
 - (E) I, II, and III

4. In the xy -plane, the line that is tangent to the graph of $y = x^2$ at the point $(2, 4)$ has a slope of
 - (A) $\frac{1}{2}$
 - (B) 1
 - (C) 2
 - (D) 4
 - (E) 8

5. The set $\{1, 2, 4, 7, 8, 11, 13, 14\}$ forms a group under the operation of multiplication modulo 15. Which of the following is the cyclic subgroup generated by $\{7\}$?
 - (A) $\{1, 7\}$
 - (B) $\{1, 2, 7, 14\}$
 - (C) $\{1, 4, 7, 13\}$
 - (D) $\{1, 7, 8, 13, 14\}$
 - (E) $\{4, 7, 11, 14\}$

6. For each real number $t \neq 0$, define the function $\phi_t : \mathbb{R} \rightarrow \mathbb{R}$ by $\phi_t(x) = |x|^{|t|}$. A subset A of real numbers is called invariant with respect to the collection of functions ϕ_t if $\phi_t(A) \subseteq A$ for each $t \neq 0$. For this collection of functions, which of the following intervals are invariant?
 - I. $(0, 1]$
 - II. $[0, \frac{1}{2})$
 - III. $(0, \infty)$
 - (A) I only
 - (B) II only
 - (C) III only
 - (D) I and III
 - (E) II and III



7. A portion of the graph of a continuous nonnegative function $y = \psi(x)$ is shown above, where

$$\psi(0) = 0 \text{ and } \psi\left(\frac{1}{n}\right) = 0 \text{ for each positive}$$

integer n . If the graph of $y = \psi(x)$ between $x = \frac{1}{n+1}$ and $x = \frac{1}{n}$ consists of the congruent sides of an isosceles triangle of height 1 for each positive integer n , then $\int_0^1 \psi(x) dx =$

- (A) $\frac{1}{2}$
 (B) $\frac{1}{4}$
 (C) $\frac{1}{\pi}$
 (D) $\frac{1}{e}$
 (E) $\frac{2}{e}$

8. The function f is differentiable on the interval $0 < x < 4$. If $f(1) = 1$ and $f(3) = 7$, then for some $1 < c < 3$, $f'(c)$ must be equal to

- (A) 1
 (B) 2
 (C) 3
 (D) 4
 (E) 6

9. If a and b are integers, how many matrices of the form $\begin{bmatrix} 2 & a \\ b & 3 \end{bmatrix}$ are not invertible?

- (A) One
 (B) Two
 (C) Four
 (D) Eight
 (E) More than eight

10. If V_n is the real vector space of all n -tuples of real numbers for each $n > 1$, which of the following must be true?

- I. Every basis of V_n contains exactly n vectors.
 II. Every basis of V_n is an orthogonal set of vectors.
 III. Every set of $n + 1$ vectors of V_n is a linearly dependent set.

- (A) I only
 (B) II only
 (C) I and II
 (D) I and III
 (E) II and III

ANSWER KEY

1. A
 2. E
 3. B
 4. D
 5. C
 6. D
 7. A
 8. C
 9. D
 10. D