

Statistical Analysis of Egyptian Skull Measurements

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Overview

Our data set consists of four measurements of 30 Egyptian skulls taken from each of five different time periods. These measurements are: maximal breadth of skull, basibregmatic height of skull, basialveolar length of skull, and nasal height of skull. All measurements were taken in millimeters. We hypothesize that the skull measurements vary over time, due to the influence of immigration.

For the majority of the report, we will focus on the data from the maximal breadth measurement. For each of the five time periods, we will compare the results from samples of size 30 to the results from subsets of these samples of size ten. This will demonstrate limitations inherent to smaller sample sizes. At the end of the report, we will examine some general trends in all four measurements over the duration of the time periods, using only the larger samples of size 30.

Analysis of Maximal Breadth Data

Descriptive Statistics

A summary of the descriptive statistics (mean, variance, and standard deviation) of the maximal breadth of the skulls appears below in Table 1. A histogram with normal curve representing these data for each time period can be found in Figures 1-5.

Table 1: Descriptive Statistics for Maximal Breadth of Skull, Sample Size = 30

Year	Sample Mean	Sample Variance	Sample Standard Deviation
4000 BCE	131.37	26.32	5.13
3300 BCE	132.37	23.14	4.81
1850 BCE	134.47	12.11	3.48
200 BCE	135.50	15.37	3.92
150 CE	136.17	28.62	5.35

4000 BCE Maximal Breadth Sample Size = 30

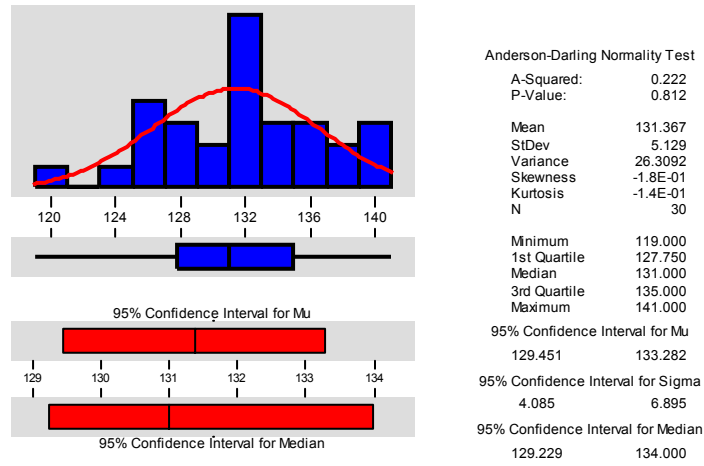


Figure 1

3300 BCE Maximal Breadth Sample Size = 30

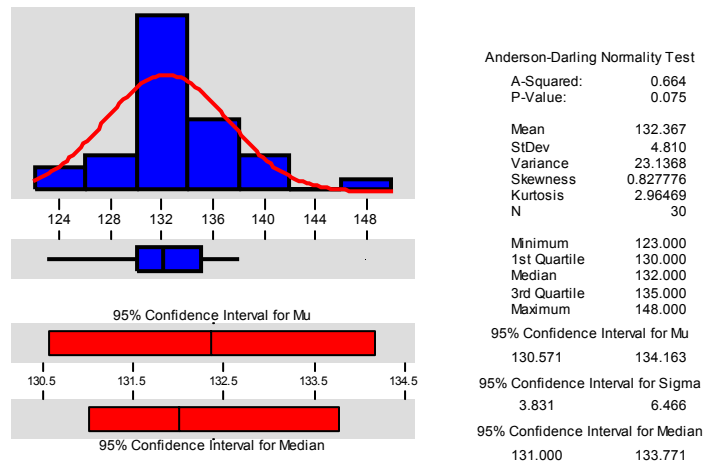


Figure 2

1850 BCE Maximal Breadth Sample Size = 30

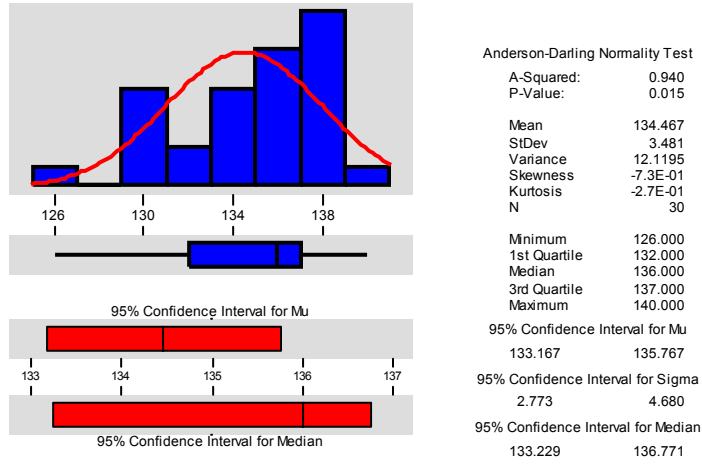


Figure 3

200 BCE Maximal Breadth Sample Size = 30

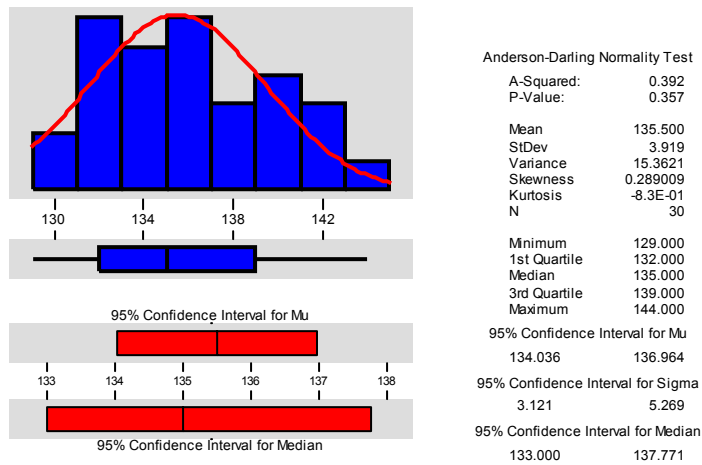


Figure 4

150 CE Maximal Breadth Sample Size = 30

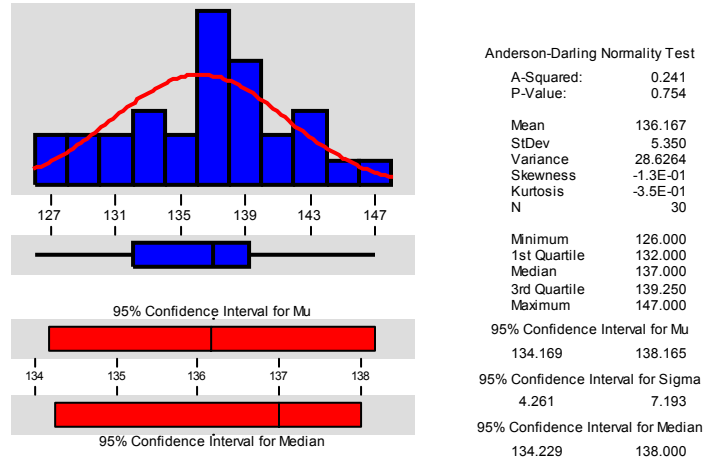


Figure 5

Table 2: Descriptive Statistics for Maximal Breadth of Skull, Sample Size = 10

Year	Sample Mean	Sample Variance	Sample Standard Deviation
4000 BCE	130.30	38.19	6.18
3300 BCE	132.20	15.52	3.94
1850 BCE	135.40	8.07	2.84
200 BCE	135.10	12.96	3.60
150 CE	134.50	22.28	4.72

4000 BCE Maximal Breadth Sample Size = 10

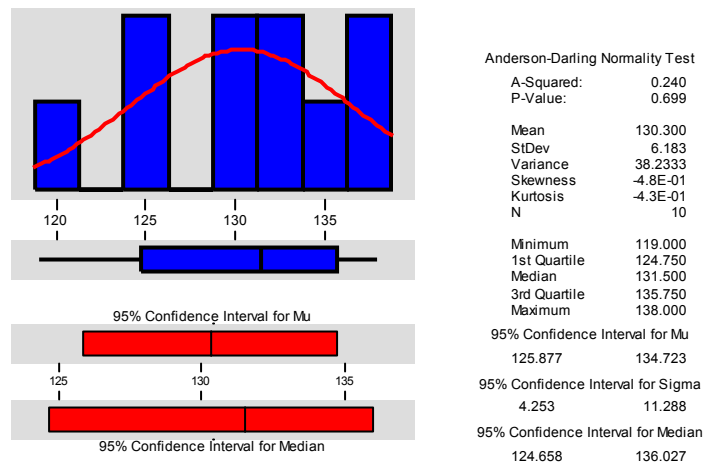


Figure 6

3300 BCE Maximal Breadth Sample Size = 10

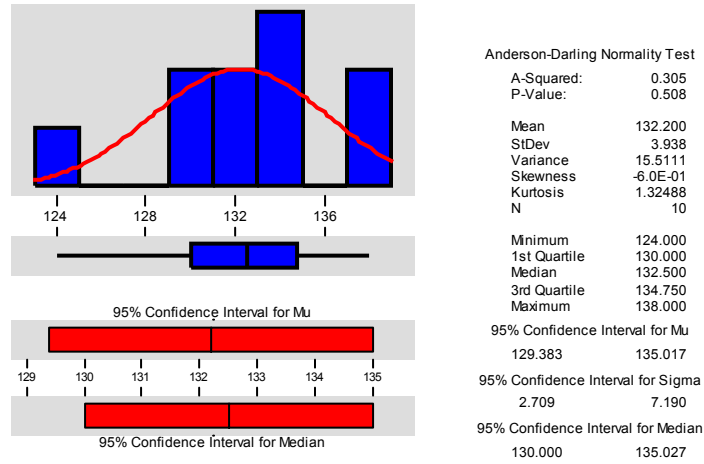


Figure 7

1850 BCE Maximal Breadth Sample Size = 10

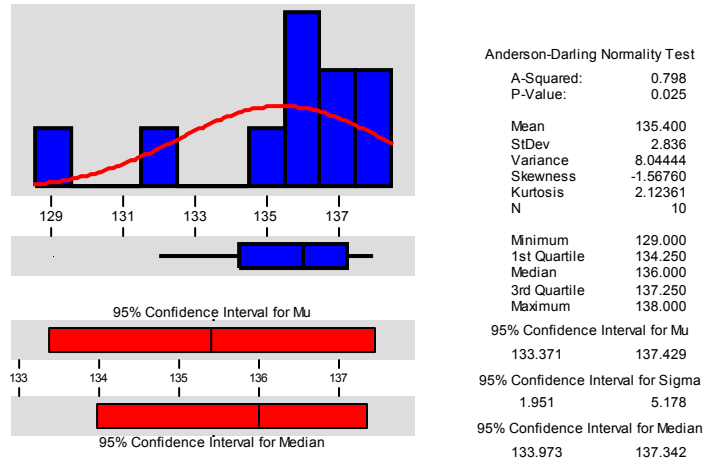


Figure 8

200 BCE Maximal Breadth Sample Size = 10

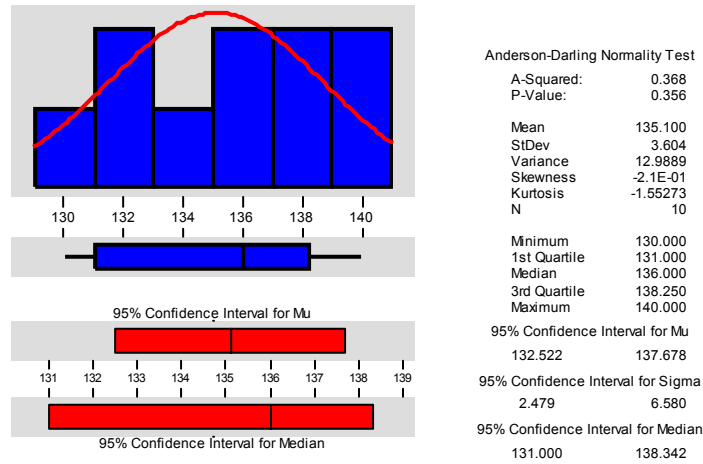


Figure 9

150 CE Maximal Breadth Sample Size = 10

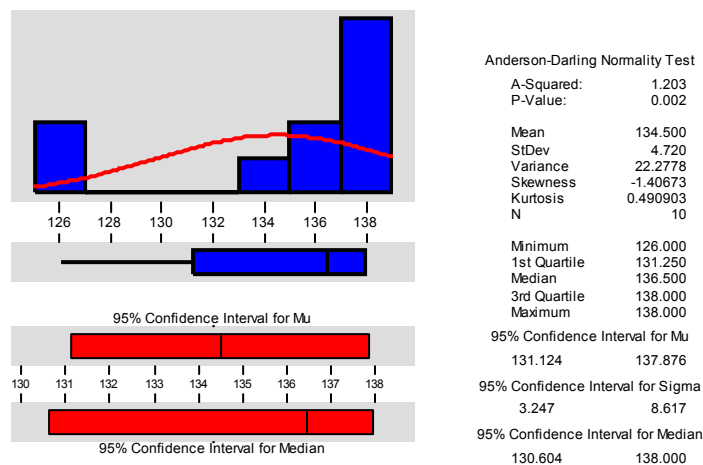


Figure 10

By comparing the data in Table 1 to the data in Table 2, we see the larger sample size yields a more consistent increase in the sample mean, and more consistency in sample variance and standard deviation than the smaller sample size. This indicates the larger sample has more reliable results than the smaller sample.

Boxplots

To compare the accuracy of the large samples ($n=30$) to the small samples ($n=10$), we compare boxplots of both the sample sizes below. Over the course of the five time periods, the boxplots for the larger sample sizes (fig. 11) reveal a definite trend of increasing maximal breadth in the skulls. The boxplots of the smaller sample sizes (fig. 12) also imply this trend, but with less certainty.

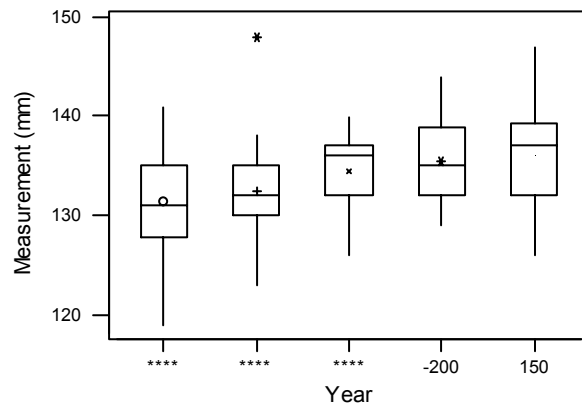


Figure 11: Boxplots of Maximal Breadth of Skull, Sample Size = 30

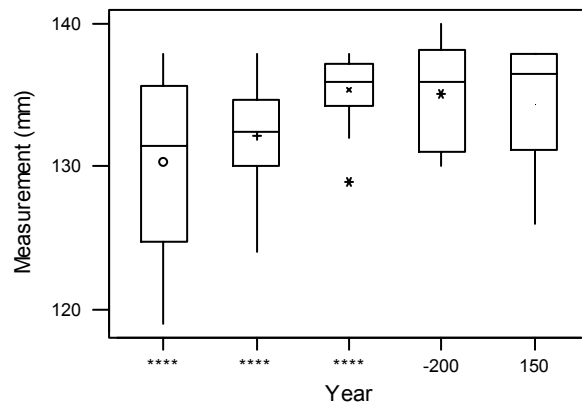


Figure 12: Boxplots of Maximal Breadth of Skull, Sample Size = 10

Confidence Intervals for Means, Variances, and Ratios of Two Variances

To examine the data further, we consider confidence intervals for the mean and variance of the large data sets, mean and variance for the small data sets, and the difference between means and the ratio of variances in the large data sets.

Table 3: Confidence Intervals for Mean, n = 30

Year	90% Lower Bound	90% Upper Bound	95% Lower Bound	95% Upper Bound	99% Lower Bound	99% Upper Bound
4000 BCE	129.83	132.91	129.53	133.21	128.95	133.78
3300 BCE	130.93	133.81	130.65	134.09	130.11	134.63
1850 BCE	133.42	135.52	133.22	135.72	132.83	136.11
200 BCE	134.32	136.68	133.54	136.40	133.66	137.34
150 CE	134.56	137.78	134.26	138.08	133.65	138.69

Table 4: Confidence Intervals for Mean, n = 10

Year	90% Lower Bound	90% Upper Bound	95% Lower Bound	95% Upper Bound	99% Lower Bound	99% Upper Bound
4000 BCE	126.72	133.88	125.85	134.75	123.95	136.65
3300 BCE	129.91	134.49	129.36	135.04	128.14	136.26
1850 BCE	133.75	137.05	133.36	137.44	132.47	138.33
200 BCE	133.01	137.19	132.51	137.69	131.39	138.81
150 CE	131.76	137.24	131.10	137.90	129.64	139.36

As expected, the large sample size (n=30) is more accurate in determining the mean than the small sample size (n=10). This is illustrated in Tables 3 and 4 by the fact that the confidence intervals are larger for the small sample size.

Table 5: Confidence Intervals for Variance, n = 30

Year	90% Lower Bound	90% Upper Bound	95% Lower Bound	95% Upper Bound	99% Lower Bound	99% Upper Bound
4000 BCE	17.94	43.16	16.58	47.57	14.48	58.17
3300 BCE	15.77	37.95	14.58	41.88	12.73	51.14
1850 BCE	8.23	19.86	7.63	21.92	6.66	26.76
200 BCE	10.45	25.21	9.68	27.82	8.45	33.97
150 CE	19.47	46.95	18.04	51.82	15.75	63.27

Table 6: Confidence Intervals for Variance, n = 10

Year	90% Lower Bound	90% Upper Bound	95% Lower Bound	95% Upper Bound	99% Lower Bound	99% Upper Bound
4000 BCE	20.24	103.37	18.07	127.17	14.51	198.21
3300 BCE	8.23	42.06	7.29	51.68	5.90	80.55
1850 BCE	4.28	21.87	3.79	26.87	3.07	41.88
200 BCE	6.87	35.12	6.09	43.16	4.93	67.26
150 CE	11.81	60.38	10.47	74.19	8.47	115.63

Again, as expected, the large sample size (n=30) is more accurate in determining the variance than the small sample size (n=10). This is illustrated in Tables 5 and 6 by the fact that the confidence intervals are larger for the small sample size.

Table 7: Difference Between Means, n = 30

Year	90% Lower Bound	90% Upper Bound	95% Lower Bound	95% Upper Bound	99% Lower Bound	99% Upper Bound
4000 BCE and 150 CE	-7.02	-2.58	-7.45	-2.15	-8.28	-1.32

The data reflect that with at least 99% confidence, we can state that the skull sizes have increased from 4000 BCE to 150 CE.

Table 8: Ratio of Two Variances, n = 30

Year Ratios	90% Lower Bound	90% Upper Bound	98% Lower Bound	98% Lower Bound
4000 BCE 3300 BCE	0.61	2.10	0.48	2.70
4000 BCE 1850 BCE	1.18	3.99	0.91	5.19
4000 BCE 200 BCE	0.93	3.15	0.72	4.09
4000 BCE 150 CE	0.50	1.69	0.39	2.19

In the first row of Table 8, we see the variance of measurements of skulls from 4000 BCE is probably larger than the variance for skulls from 3300 BCE. This could be interpreted to mean that the population is becoming more homogeneous genetically, since decreased immigration into a region would lead to the development of more consistent genetic characteristics throughout the population. This trend toward smaller variance continues through 1850 BCE, then reverses as we look at the statistics for 200 BCE and 150 CE. By 150 CE, it appears the population is nearly as diversified as it was in 4000 BCE.

Table 9: Ratio of Two Variances, n = 10

Year Ratios	90% Lower Bound	90% Upper Bound	98% Lower Bound	98% Lower Bound
<u>4000 BCE</u> 3300 BCE	0.77	7.82	0.46	13.16
<u>4000 BCE</u> 1850 BCE	1.48	15.04	0.88	25.31
<u>4000 BCE</u> 200 BCE	0.93	2.73	0.55	15.73
<u>4000 BCE</u> 150 CE	0.54	5.44	0.32	9.15

As usual, the small data set in Table 9 reflects less accuracy, but the same trend as the large set.

Analysis of Other Skull Measurements

This set of data includes other skull measurements. Like the maximal breadth measurements, the nasal height appears to be increasing for each time period.

Basialveolar length and basibregmatic height, on the other hand, seem to be decreasing. Sample variance and sample standard deviation do not appear to fit an overall trend. This tells us we really don't have enough information to examine trends in variance throughout the time periods considered.

Table 10: Descriptive Statistics for Basialveolar Length, Sample Size = 30

Year	Sample Mean	Sample Variance	Sample Standard Deviation
4000 BCE	99.17	34.57	5.88
3300 BCE	99.07	18.92	4.35
1850 BCE	96.03	20.70	4.55
200 BCE	94.53	21.07	4.59
150 CE	93.50	25.60	5.06

Table 11: Descriptive Statistics for Basibregmatic Height, Sample Size = 30

Year	Sample Mean	Sample Variance	Standard Deviation
4000 BCE	133.60	19.98	4.47
3300 BCE	132.70	21.62	4.65
1850 BCE	133.80	24.80	4.98
200 BCE	132.30	26.32	5.13
150 CE	130.33	24.70	4.97

Table 12: Descriptive Statistics for Nasal Height, Sample Size = 30

Year	Sample Mean	Sample Variance	Standard Deviation
4000 BCE	50.53	7.62	2.76
3300 BCE	50.23	8.76	2.96
1850 BCE	50.57	12.60	3.55
200 BCE	51.97	7.95	2.82
150 CE	51.37	13.84	3.72

